THE ELEMENTS OF EUCLID,

Explained and Demonstrated in a New and most easie Method.

With the USES of each

PROPOSITION

In all the Parts of the

MATHEMATICKS.

By Claude Francois Milliet D' Chales. *a Jesuite.*

Done out of FRENCH, Corrected and Augmented, and Illustrated with Nine Copper Plates, and the Effigies of *EUCLID*, By *Reeve Williams*, Philomath.

LONDON:

Printed for *Philip Lea*, Globemaker, at the *Atlas* and *Hercules* in the *Poultrey*, near *Cheapside*, 1685.

To the Honourable

Samuel Pepys, Esq;

Principal Officer

OF THE

NAVY,

SECRETARY

TO THE

ADMIRALTY,

AND

PRESIDENT

OF THE

ROYAL SOCIETY.

Honoured Sir,

THE Countenance and *Encouragment, you have always given to* Mathematical Learning, *especially*

Halliday 1801

especially as it hath a tendency to Promote the Publique Good; *has emboldened me, humbly to Present your Honour, with this little Peice; which hath the Admirable* Euclid *for its Author, and the Learned* D'Chales *for the Commentator, the excellency of the Subject, with the Apt and Profitable Application thereof, in its Uses; did first induce me to Translate it for my own use, the benefit and quickening in those* Mathematical Studies, *that some*

ſome profeſſed to have receiv-ed, did prevail with me to make it Publique, and the great Obligations *I lie under, from the many undeſerved Favours of your Honour, toward me; I thought did engage me, on this occaſion, to make ſome Publique Teſtimony and Acknowledgment thereof. I therefore Humbly beg your Honours Patronage of this little Book, and your Pardon for this Addreſs, intreating you will be pleaſed to look upon it with that Benign Aſpect,*

as

Dedication.

as you have been pleaſed always to vouchſafe to him who is

Your Humble

and moſt

Obliged

Servant

REEVE WILLIAMS.

THE

Authors Preface

TO THE

READER.

HAving long ſince obſerved, that the greateſt part of thoſe that learn *Euclid*'s *Elements*, are very often diſſatisfied therewith, becauſe they know not the uſe of *Propoſitions* ſo inconſiderable in appearance; and yet ſo difficult: I thought it might be to good purpoſe, not only to make them as eaſie as poſſible, but alſo to add ſome *Uſes* after each *Propoſition*, to ſhew how they are applicable to Practice. And this hath obliged me to change ſome of the *Demnoſtrations*, which I looked upon to be too troubleſome, and above the uſual reach

reach of beginners, and to ſubſtitute others more intelligible. For this cauſe, I have Demonſtrated the *Fifth Book* after a method much more clear than that, that makes uſe of *equimultiples*. I have not given all the *Uſes* of the *Propoſitions*, for that would have made it neceſſary to recite all the *Mathematicks*, and would have made the Book too big, and too hard. Wherefore, I have only made choice of ſome of the plaineſt and eaſieſt to conceive, I would not have you to ſtand too much upon them, nor make it your ſtudy to underſtand them perfectly becauſe they depend upon other principles beſides, for which Reaſon I have diſtinguiſhed them with the *Italick* Letter. This is the deſign of this ſmall Treatiſe, which I willingly publiſh in a time when the *Mathematicks* are more than ever ſtudied.

Milliet D' chales

THE TRANSLATOR'S PREFACE.

THE *Reader having perused the Authors Preface, with this farther intimation, that he will find the Subject and Scope of this work Succinctly and pertinently presented to him, in the Argument before each particular book, may I presume, expect the less from me, and I shall not at all endeavor to bespeak the Readers acceptance of* Euclid's *Elements, or persuade him to beleive, the Necessary and Excellent Usefulness thereof, because every mans Experience, so far as he understands them, is an abundant Testimony thereunto. Neither shall I need to commend the Method with the uses of our Author* D' Chales, *(who is well known to the learned of this Age by his*

his Several excellent Mathematical Tracts (* for whosoever shall be a while conversant with this book, may I presume find that instruction and incourgment in the learning of Euclid's Eliments as he hath not before met with in our English Tongue, And this as it hath been my own experience, since I Translated it from the French for the use of my English Schollars, so it is one great cause of its coming abroad into the World: for such as had Learned by it found it difficult to attain, unless Transcribed, which they thought tedious, being a subject so Voluminous in Manuscript, and full of Schemes, I did therefore at their request, and the importunity of some Friends condescend to the Printing hereof, though not without much averseness to my own mind, being unwilling to Expose my self in any Publick thing, in this nice Critical Age; But that difficulty being now overcome, I shall only give the Reader to understand, that I have faithfully rendred this Piece into English according to the sense of the Author, but here and there omitting some small matters, which I judged not so properly related to the subject of this Work, and therein will make no want to the Reader, nor I hope be any offence to the Ingenious Author himself, I have

* Cursus Mathe his Navigation, his Local Motion, &c.

have only one thing more to add, and that upon the account of an Objection I have met with, that here is not all the Books of Euclid, *and it is true, here is not all, here are only the first six Books, and the Eleventh, and Twelfth, the other being purposely omitted by our learned Author, who judged the understanding of these to be sufficient, for all the parts of the Mathematicks,* * *and I could also give Instances of other excellent Authors, that are of his Opinion, and have taken the like course, nay the truth is some very learned in the Mathematicks, have reduced the Propositions of these Books to a much lesser Number, and yet have thought them a compleat foundation to all the Sciences Mathematical, but I shall not trouble my Reader farther on this account, not doubting but when he hath perused and well considered our* Euclid, *he will have a better Opinion thereof, then any thing I can now say, may Justly hope to beget in him, and so I shall submit my whole Concernment herein to the Impartial Reader, and remain ready to Serve him.*

* See Argument before the Eleventh Book, Page 304.

Reeve Williams.

Form my School at the
Virginia Coff-house
in St. *Michaels Alley*
in *Cornhill.*

- *A. D.*

ADVERTISEMENT.

WHereas in the *French*, all the Definitions which needed all the Propoſitions, as alſo all thoſe uſes, our Author thought fit to Illuſtrate by Schemes, were done in the Book in wood, here they are in Copper plates to be placed at the end of the Book. The Definitions and uſes are in the Plates marked with Arithmetical Characters or Ziphers; but the Propſitions in Alphabetical Ziphers at the begining of the Book, or at leaſt when you come to the firſt Definition, you are referred to the number of the Plate, in which you ſhall find the Scheme proper thereto, as alſo the Def Prop. and uſes belonging unto the Book, unleſs the Plate could not contain them, and then you are referred to the next Plate; whereon the head thereof you ſhall find the Book it belongeth to, and the Propoſitions, or uſes, continued in their order.

THE

EIGHT BOOKS OF Euclid's *Elements*: *With the Uses of each* PROPOSITION.

The FIRST BOOK.

EUCLID's *Design in this Book is to give the first Principles of Geometry; and to do the same Methodically, he begins with the Definitions, and Explication of the most ordinary Terms, then he exhibits certain Suppositions, and having proposed those Maxims which natural* Reason *teacheth, he pretends to put forward nothing without Demonstration, and to convince any one which will consent to*

nothing but what he shall be obliged to acknowledge; in his first Propositions he treateth of Lines, and of the several Angles made by their intersecting each other, and having occasion to Demonstrate their Proprieties, and compare certain Triangles, he doth the same in the First Eight Propositions; then teacheth the Practical way of dividing an Angle and a Line into two equal parts, and to draw a Perpendicular he pursues to the propriety of a Triangle; and having shewn those of Parallel Lines, he makes an end of the Explication of this First Figure, and passeth forwards to Parallelograms; giving a way to reduce all sorts of Polygons into a more Regular Figure: He endeth this Book with that Celebrated Proposition of Pythagoras, *and Demonstrates, that in a Rectangular Triangle, the Square of the Base is equal to the sum of the Squares of the Sides, including the Right Angle.*

DEFINITIONS.

1. A Point is that which hath no part.

This Definition is to be understood in this sence. The quantity which we conceive without distinguishing its parts, or without thinking

thinking that it hath any, is a Mathematical Point far differing from those of Zeno, *which were altogether indivisible, since one may doubt with a great deal of Reason, if those last be possible, which yet we cannot of the first, if we conceive them as we ought.*

2. A Line is a length without breadth.

The sense of this Definition is the same with that of the foregoing, the quantity which we consider, having length without making any reflection on its breadth, is that we understand by the word Line, although one cannot draw a real Line which hath not a determinate breadth, it is generally said that a Line is produced by the motion of a Point, which we ought well to take notice of, seeing that by a motion after that manner may be produced all sorts of quantities; imagine then that a Point moveth, and that it leaveth a trace in the middle of the way which it passeth, the Trace is a Line.

3. The Two ends of a Line are Points.

4. A streight Line is that whose Points are placed exactly in the midst, or if you would rather have it, a streight Line is the shortest of all the

Lines which may be drawn from one Point to another.

5. A Superficies is a quantity to which is given length and breadth without considering the thickness.

6. The extremities of a Superficies are Lines.

7. A plain or straight Superficies is that whose Lines are placed equally between the extremities, or that to which a streight Line may be applyed any manner of way.

Plate I. Fig. 1.

I have already taken notice that motion is capable of producing all sorts of quantity, whence we say that when a Line passeth over another, it produces a superficies or a Plain, and that that motion hath a likeness to Arithmetical Multiplication; imagine that the Line A B moveth along the Line B C, keeping the same situation, without inclining one way or the other, the Point A shall describe the Line A D, the Point B the Line B C; and the other Points between other Parallel Lines, which shall compose the Superficies A B C D. I add, that this motion corresponds with Arithmetical Multiplication, for if I know the number of Points which are in the Lines A B, B C, Multiplying of them one by the other, I

shall

ſhall have the number of Points which Compoſeth the Superficies A B C D; as if A B contains four points, and B C ſix; ſaying, Four times Six are Twenty Four, the Superficies A B, C D, ſhould be Compoſed of Twenty Four Points: Now I may take for a Mathematical Point any quantity whatſoever, for Example a Foot, provided I do not ſubdivide the ſame into Parts.

8. A plain Angle is the opening of Two Lines which interſect each other, and which Compoſe not one ſingle Line.

As the opening D, of the Lines A B, C B; which are not parts of the ſame Line. Fig. 2.

A Right Lined Angle is the opening of two ſtreight Lines.

It is principally of this ſort of Angles which I intend to treat of at preſent, becauſe experience doth make me perceive, that the moſt part of thoſe who begin, do miſtake the meaſuring the quantity of an Angle, by the length of the Lines which Compoſeth the ſame.

The moſt open Angle is the greateſt, that is to ſay, when the Lines including an Angle are farther aſunder than thoſe of another Angle, taking them at the ſame diſtance from the Points of interſection of Fig. 3, 4.

their Lines, the first is greater than the Second; so the Angle *A* is greater than *E*; because if we take the Points *B* and *D*, as far distant from the Point *A*, as the Points *G* and *L* are from the Points *E*; the Points *B* and *D* are farther asunder than the Points *G* and *L*; from whence I conclude, that if *EG*, *EL*, were continued, the Angle *E* would be of the same Measure, and less than the Angle *A*.

We make use of Three Letters to express an Angle, and the Second Letter denotes the Angular Point, as the Angle *B A D*, is the Angle which the Lines *B A*, *A D*, doth form, at the Point *A*; the Angle *B A C*, is that which is formed by the Lines *B A*, *A C*; the Angle *C A D* is comprehended under the Line *C A*, *A D*.

Fig. 3. The Arch of a Circle is the measure of an Angle, thus designing to measure the quantity of the Angle *B A D*, I put one Foot of the Compasses on the Point *A*, and with the other I describe an Arch of a Circle *B C D*, the Angle shall be the greater by how much the Arch *B C D*, which is the measure thereof, shall contain a greater portion of a Circle, and because that commonly an Arch of a Circle is divided into Three Hundred and Sixty equal Parts,

Parts, called Degrees: It is said that an Angle containeth Twenty, Thirty, Forty Degrees, when the Arch included betwixt its Lines, contains Twenty, Thirty, Forty Degrees; so the Angle is greatest, which containeth the greatest number of Degrees: As the Angle B A D, is greater than G E L, the Line C A divideth the Angle B A D, in the middle, because the Arches BC, C D, are equal, and the Angle BAC is a part of B A D, because the Arch B C is part of the Arch B D.

10. When a Line falling on another Line, maketh the Angle on each side thereof equal: Those Angles are Right Angles, and the Line so falling is a Perpendicular.

As if the Line AB falling on C D, makes Fig. 5. *the Angles A B C, A B D, equal, that is if having on B as a Center described a Semicircle C A D, the Arches A C, D A are found equal, the Angle A B C, A B D, are called Right Angles, and the Line A B a Perpendicular. Now because the Arch C A D is a Semicircle, the Arch C A, A D, are each of them a Quadrant, that is to say the Fourth part of the Three Hundred and Sixty Degrees, which is Ninety.*

11. An Obtuse Angle is greater than a Right Angle.

Fig. 5. *As the Angle E B D is Obtuse or Blunt, because its Arch E A D contains more than a Quadrant.*

12. An Acute Angle is less than a Quadrant.

Fig. 5. *As the Angle E B C is Acute, because the Arch E C, which is the measure thereof, is less than Ninety Degrees.*

13. A Term is the extremity or end of a Quantity.

14. A Figure is a Quantity termined by one or many Terms.

It ought to be bounded and inclosed on every side to be called a Figure.

15. A Circle is a Plain Figure, whose bounds are made by the winding or turning of a Line, which is called Circumference, and which is equally distant from the middle Point.

Fig. 6. *The figure R V S X is a Circle, because all the Lines T R, T S, T V, T X, drawn from the Point T, to the Line R V S X, are equal.*

16. This middle Point is called the Center.

Fig. 7. 17. The Diameter of a Circle is any Line whatsoever which passeth through the Center, and which ends at the Circumference,

cumference, cutting the same into two equal parts.

As the Line V T X, or R T S; some one may perhaps doubt whether the Line V T X doth effectually divide the Circle into Two equally, that is, whether the part V S X, be equal to the part V R X; that they are, I thus prove,

Let it be imagined that the part V R X, be placed on V S X, I say that the one shall not exceed the other; for if the one as V S X, should exceed the other V R X, the Line T R would then be shorter than T S, the same of T Y, in respect of T Z, which is contrary to the Definitions of a Circle; that is, that all the Lines drawn from the Center of a Circle to the Circumference, be all equal. Fig. 7.

18. A Semicircle is a Figure Termined by the Diameter and half the Circumference, as V S X. Fig. 7.

19. Right Lined Figures are Termined by streight Lines, there are some of Three, of Four, of Five, or as many sides as you would have them.

Euclide *distinguisheth Triangles by their Angles, or by their sides.*

20. An Equilater Triangle is that whose Three sides are equal, *as A B C.* Fig. 8.

Fig. 8. 21. An Isosceles Triangle is that which hath two equal Sides.

As if the Sides A B, A C, are equal, the Triangle A B C is an Isosceles Triangle.

Fig. 9. 22. Scalenum is a Triangle whose three Sides are unequal, *as D E F.*

Fig. 9. 23. A Right Angled Triangle is that which hath one Angle right, *as D E F, supposing the Angle E to be right.*

Fig. 10. 24. An Amblygonium Triangle hath one Angle Obtuse, *as G H I.*

Fig. 10. 25. An Oxygonium Triangle hath all its Angles Acute, *as A B C.*

Fig. 8. 26. A Rect-Angle is a figure having four Sides, which hath all its Angles right.

Fig. 11. 27. A Square hath all its Sides equal and all its Angles right, *as A B.*

Fig. 12. 28. A Rect-Angular figure whose Lines are unequal and Angles right, is called an Oblong or long Square, *as C D.*

Fig. 13. 29. A Rhombus or Diamond figure, is that which hath four equal Sides, but is not right Angled, *as E F.*

Fig. 14. 30. A Rhomboides or Diamond like figure, is that whose opposite Sides and opposite Angles are equal, but hath neither equal Sides nor Right Angles, *as G H.*

31. Other

31. Other Irregular figures having four ſides, are called Trapeziums.

32. Parallel or Equidiſtant Right Lines, are ſuch which being in the ſame Superficies, if infinitely produced would never meet, *as the two Lines A B, C D.* Fig. 15.

33. A Parallelogram is a figure whoſe oppoſite Sides are parallel, *as the figure A B C D, of which the Sides A B, C D, AC, B D, are parallel.* Fig. 15.

34. The Diameter of a Parallelogram is a ſtreight Line drawn from Angle to Angle, *as B C.* Fig. 16.

35. The Complements are the two leſſer Parallelograms, through which the Diameter paſſeth not, *as A F E H, G D I E.* Fig. 16.

Poſtulata *or* SUPPOSITIONS.

1. IT is ſuppoſed that a ſtreight Line may be drawn from any Point whatſoever, to any other Point.

2. That a ſtreight Line may be infinitely continued.

3. That on a Center given a Circle may be deſcribed, with any diſtance taken between the Compaſſes.

AXI-

AXIOMS.

1. QUantities or things equal to the ſame third, are alſo equal among themſelves.

2. If to two equal quantities be added two other Quantities which are alſo equal, thoſe Quantities which are produced ſhall be equal.

3. If from two equal Quantities be taken away two other equal Quantities, thoſe Quantities remaining ſhall be equal.

4. If equal Quantities be added to unequal Quantities, their ſums ſhall be unequal.

5. If from equal Quantities be taken away unequal Quantities the Quantities remaining ſhall be unequal.

6. The Quantities which are double, treble, quadruple, to the ſame, are equal among themſelves.

7. Quantities are equal, when applyed together, the parts of the one are not extended beyond thoſe of the other.

8. Lines and Angles are equal, when the one being applied to the other they agree.

9. Every

9. Every whole is greater than its part, and equal to all its parts taken together.

10. All Right Angles are equal amongst themselves.

Let there be proposed two Right Angles Fig. 17.
A B C, E F H, I say they are equal to each other; for if two equal Circles be described, C A D, H E G, on the Centers B and F, the quarters of Circles C A, H E, which are the measures of the Angles, A B C, E F H, shall be equal; therefore the Angles A B C, E F H, whose measures being equal, are consequently equal.

The Eleventh maxim of *Euclid* Fig. 18.
beareth this signification, that if the Lines A B, C D, maketh with the Line E F, which cutteth both the said Lines, the Internate Angle B E F, D F E, less than two Right Angles, the Lines A B, C D, being continued, shall meet each other on the same side with B and D.

Although this maxim be true, yet it is not evident enough to be received for a maxim, wherefore I substitute another in the place thereof.

11. If two Lines are parallel, all the perpendiculars inclosed between them shall be equal.

Fig. 19. *As if the Line A B, C D, are parallels, the perpendicular Lines F E, H G, are equal, for if E F was greater than G H, the Lines A B and C D should be farther distant from each other towards the Points E and F, than towards G and H; which would be contrary to the Definition of parallels, which sayeth that they are equidistant from each other.*

12. Two Right Lines do not comprehend a Superficies, that is to say, do not incompass a space on every side.

13. The streight Lines have not the same common Segment.

Fig. 20. *I would say, that of two streight Lines A B, C D, which meeteth each other in the Point B, is not made one single Line B D, but that they cut each other, and separate after their so meeting.*

For if a Circle be described on the Center B, A F B shall be a Semicircle; seeing the Line A B D passing through the Center B, divideth the Circle into two equally; the Segment C F D should be also a Semicircle, if C B D were a streight Line, because it passeth through the Center B; therefore the Segment C F D should be equal to the Segment A F D, a part as great as the whole; which would be contrary to the Ninth Axiom.

A D-

ADVERTISEMENT.

WE have two ſorts of Propoſitions, ſome whereof conſidereth only a truth without deſcending to the practice thereof, and we call thoſe Theorms.

The other propoſeth ſomething to be done or made, and are called Problems.

The firſt numbers of Citations is that of the Propoſition, the ſecond that of the Book: As by the 2 *of the* 3, *is ſignified the* ſecond *propoſition of the* third *Book; but if one meet with one number, thereby is meant the Propoſition of the Book, whoſe Explication is in hand.*

PROPOSITION I.

PROBLEM.

UPon a finite Right Line AB, to deſcribe an Equilateral Triangle ACB.

From the Centers A and B, at the diſtance of AB, deſcribe the Circles cutting each other in the Point C; from whence draw two Right Lines CA, CB; thence are AC, AB, BC, AC, equal; wherefore the Triangle ACB is equilateral, which was to be done.

USE.

USE.

Euclid has not applyed this Proposition to any other use, but to demonstrate the two following Propositions, but we may apply it to the measuring of an inaccessible Line.

Use. 1. *As for Example, let A B be an inaccessable Line, which is so by reason of a River or some other Impediment, make an Equaliteral Triangle, as B D E, on Wood, or Brass, or on some other convenient thing, which having placed Horizontally at a station at B, look to the Point A, along the Side B D, and to some other Point C, along the side B E; then carry your Triangle along the Line B C, so far, that is untill such time as you can see the Point B, your first station by the side C G, and the Point A by the Side C E: I say that then the Lines C B and C A are equal; wherefore, if you measure the Line B C, you will likewise know the length of the Line A B.*

PRO-

PROPOSITION II.

PROBLEM.

AT a Point given A, to make a Right Line AC, equal to a Right Line given BC.

From the Center C, at the distance CB, describe the Circle CBE, joyn AC; upon which raise the Equilateral Triangle ADC, produce DC to E, from the Center D, and the distance DE; describe the Circle DEH. Let DA be produced to the Point G in the Circumference thereof, then AG is equal to CB; for DG is equal to DE, and DA to DC; wherefore AG, CE, BC, AG, are equal; which was to be done.

SCHOL.

THE Line AG might be taken with a pair of Compasses; but the so doing answers no Postulate, as Proclus *well intimates.*

PRO-

PROPOSITION III.

PROBLEM.

TWo Right Lines, A and BC, being given, from the greater BC, to take away the Right Line BE, equal to the leſſer A.

At the Point B, draw the Right Line BD equal to A, the Circle deſcribed, from the Center B, at the diſtance BD, ſhall cut off BE, equal to BD, equal to A, equal to BE; which was to be done.

The uſe of the two preceding Propoſitions are very evident, ſince we are obliged very often in our Geometrical Practices to draw a Line equal to a Line given, or to take away from a greater Line given, a part equal to a leſſer.

PROPOSITION IV.

THEOREM.

IF Two Triangles ABC, EDF, have two Sides, of the one BA, AC, equal to two Sides of the other ED, DF, each to his correſpondent ſide; that is BA to ED, and AC to DF; and have the Angle A equal to the Angle

D,

D, contained under the equal Right Lines, they ſhall have the Baſe B C equal to the Baſe E F, and the Triangle B A C ſhall be equal to the Triangle E D F; and the remaining Angles B, C, ſhall be equal to the remaining Angles E, F; each to each, which are ſubtended by the equal Sides.

If the Point D be applied to the Point A, and the Right Line D E placed on the Right Line A B, the Point E ſhall fall upon B, becauſe D E is equal to A B; alſo the Right Line D F ſhall fall upon A C; becauſe the Angle A is equal to D: Moreover the Point F ſhall fall on the Point C, becauſe A C is equal to D F: Therefore the Right Lines E F, B C, ſhall agree, becauſe they have the ſame terms, and ſo conſequently are equal; wherefore the Triangle B A C is equal to D E F, and the Angles B, E; as alſo the Angles C, F do agree, and are equal, which was to be demonſtrated.

USE.

Suppoſe I was to meaſure the inacceſſible Line A B, I look from the Point C to the Point A and B, then I meaſure the Angle C thus; I place a Board or Table Horizontally, Uſe 4.

Horizontally, and looking successively with a Ruler towards the Points A and B; I draw two Lines making the Angle A C B, then I measure the Lines A C and B C, which I suppose to be accessible; I turn about my Board or Table towards some other place in the Field, placing it again Horizontally at the Point F, and looking along those Lines I have drawn on my Table, I make the Angle D E F equal to the Angle C; I make also F D, F E, equal to C A, C B: Now according to this Proposition the Lines A B, D E, are equal, wherefore whatsoever the Length of the Line D E is found to be, the same is the measure of the inaccessible Line A B.

Another USE may be this.

Use 4. *Suppose you were at a Billiard Table, and you would strike a Ball B by reflection, with another Ball A: Admit C D be one Side of the Table, now imagine a perpendicular Line B D E; I take the Line D E equal to D B; I say, that if you aim and strike your Ball A directly towards the Point E, the Ball A meeting the Side of the Table in F, shall reflect from thence to B; for in the Triangle B F D,*

B F D, E F D, the Side F D is common, and the Sides B D, D E, equal, the Angles B F D, E F D equal, by the Propoſition, the Angles A F C, D F E, being oppoſite are alſo equal, as I ſhall demonſtrate hereafter; therefore the Angle of incidence A F C, is equal to the Angle of reflection B F D, and by conſequence the reflection will be from A F to F B.

PROPOSITION V.

THEOREM.

IN every Iſoſceles Triangle, the Angles which are above the Baſe are equal, as alſo thoſe which are underneath.

Let ABC be an Iſoſceles Triangle, *viz.* that the ſide A B, A C, be equal, I ſay that the Angle A B C, A C B, are equal, as alſo the Angles G B C, H C B, which ly under the Baſe B C: Let there be imagined another Triangle D E F, having an Angle D equal to the Angle A; and the Sides D E, D F, equal to A B, A C: Now ſince the Sides A B, A C, are equal, the Four Lines A B, A C, D E, D F, ſhall be equal.

Demonſtration, becauſe the Sides A B, D E, A C, D F, are equal, as alſo

alſo the Angles A and D, if we put the Triangle D E F, on the Triangle A B C, the Line D E ſhall fall upon A B and D F, on A C and F E, on B C; (by the Fourth) therefore the Angle D E F ſhall be equal to A B C, and ſince one part of the Line D E falls on A B, the whole Line D I ſhall fall on A G; otherwiſe Two ſtreight Lines would contain a ſpace, therefore the Angle I E F ſhall be equal to the Angle G B C: Now if you ſhould turn the Triangle D E F, and make the Line D F to fall on A B, and D E, on A C; becauſe the Four Lines A B, D F, A C, D E, are equal, as alſo the Angles A and D; the Two Triangles A B C, D E F, ſhall lye exactly on each other, and the Angles A C B, D E F, H C B, I E F, ſhall be equal: Now according to our firſt Compariſon, the Angle A B C was equal to D E F, and G B C to I E F; therefore the Angles A B C, A C B, which are equal to the ſame D E F, and G B C, H C B, which are alſo equal to the ſame I E F, are alſo equal among themſelves.

I thought fit not to make use of Euclid's *Demonstration, because it being too difficult for young Learners to Apprehend, they are often discouraged to proceed.*

PROPOSITION VI.

THEOREM.

IF Two Angles of a Triangle are equal, that Triangle shall be an Isosceles Triangle.

Let the Angles A B C, A C B, of the Triangle A B C be equal; I affirm, that their opposite Sides A B, A C, are equal to each other; let the Triangle D E F, have the Base D F equal to B C, and the Angle D E F, equal to A B C; as also D E F, equal to A C B; since then that we suppose that the Angles A B C, A C B, are equal; the Four Angles A B C, A C B, D E F, D F E, are equal: Now let us imagine the Base E F be so put on B C, that the Point E fall on B; it is evident, that since they be equal, the one shall not exceed the other anywise; moreover the Angle E being equal to the Angle B, and the Angle F to the Angle C; the Line E B shall fall on B A, and F D on C A; therefore the Lines

Lines E D, F B, shall meet each other in the Point A; from whence I conclude, that the Line E D is equal to B A.

Again let us turn over the Triangle D E F, placing the Point E on C, and F on B; which must so happen, because B C is supposed equal to E F; and because the Angles F and B, E and C, are supposed equal; the Side F B shall fall on B A, and E D on C A, and the Point D on A; wherefore the Lines A C, D E, shall be equal; from whence I conclude, that the Sides A B, A C, are equal to each other, since they are equal to the same Side D E.

USE.

Use. 6. *THis Proposition is made use of, in measuring all inaccessible Lines, it is said that* Thales *was the first that measured the Height of the* Egyptian Pyramids *by their Shadows, it may easily be done by this Proposition; for if you would measure the Height of the Pyramid A B, you must wait till the Sun be elevated 45 degrees above the Horizon, (that is to say) until the Angle A C B be 45 degrees; now by the Proposition the Shadow B C is equal to the Pyramid A B,*

A B, for seeing the Angle A B C is a Right Angle, and that the Angle A C B is half Right, or 45 degrees, the Angle C A B shall also be half a Right Angle, as shall be proved hereafter: Therefore the Angles B C A, B A C, are equal, and by the Sixth the Sides A B, B C, are also equal; I might measure the same without making use of the Shadow, by going backwards from B, untill the Angle A C B be half a Right Angle, which I may know by a Quadrant: Those Propositions are made use of in Trigonometry and several other Treatises.

I Omit the Seventh, whose use is only to Demonstrate the Eighth.

PROPOSITION VIII.

THEOREM.

IF Two Triangles be equally Sided, they shall also have their opposite Angles equal.

Let the Sides G I, L T; H I, V T; G H, L V, be equal: I say that the Angle G I H shall be equal to the Angle L T V, I G H, to the Angle L; I H G, to the Angle V. From the Center H, at the distance H I, let the Circle I G be described, and from the Center G,

with the Extent G I, let the Circle H I be deſcribed.

Demonſtration. If the Baſe L V be laid on the Baſe H G, they will agree, becauſe they are equal; I further add, that the Point T ſhall fall preciſely at the Circumference of the Circle I G; ſince we ſuppoſe the Lines H L, V T are equal, it muſt likewiſe fall at the Circumference I H, ſeeing G I is equal to L T; therefore it ſhall fall on the Point I, which is the Point were thoſe Circles cut each other, but if you deny it, and ſay it ſhould fall on ſome other Point, as at O, then I ſay the Line H O (that is to ſay V T,) would be greater then H I; and the Line G O (that is to ſay L T) would be leſs than G I, which is contrary to the Hypo. from whence I conclude the Triangles do agree in all their parts; and that therefore the Angles G I H, is equal to the Angle L T V.

This Propoſition is neceſſary for the eſtabliſhing the next following; moreover when we cannot meaſure an Angle, which is made in a Solid Body, by reaſon we cannot place our Inſtruments; we take the Three Sides of a Triangle, and make another on Paper, whoſe Angles we eaſily meaſure

measure, this is very useful in Dialling, and in cutting of Stone, and Bevelling of Timber.

PROPOSITION IX.

PROBLEM.

TO Bisect or Divide into two equal parts a Right-Lined Angle given, SAT.

Take AS equal to AT, and draw the Line ST, upon which make an equilateral Triangle SVT; draw the Right Line VA, it shall Bisect the Angle.

Demonstration. AS is equal to AT, and the Side AV is common, and the Base SV equal to VT; therefore the Angle SAV is equal to TAV, which was to be done.

USE.

THis Proposition is very useful to Divide a Quadrant into Degrees, it being the same thing to Divide an Angle, or to Divide an Arch into two equal parts; for the Line AV Divideth as well the Arch ST as the Angle SAT, for if you put the Semi-Diameter on a Quadrant, you cut off an Arch of 60 Degrees, and Dividing that

Arch into Two Equal parts, you have 30 Degrees, which being again Divided into two equal parts, you have 15 Degrees. It is true, to make an end of this Division you must Divide an Arch in Three, which is not yet known to Geometricians. Our Sea Compasses are Divided into 32 Points by this Proposition.

PROPOSITION X.
PROBLEM.

TO Divide a Line given into two equal parts.

Let the Line A B be proposed to be Divided into Two equal parts, erect an Equilateral Triangle A B C, (*by the 1st.*) Divide the Angle A C B into two equal parts, (*by the* 9th.) I say the Line A B is Divided into two equal parts in the Point E: *viz.* That the Lines A E, E B, are equal.

Demonstration. The Triangles A C E, B C E, have the Side C E common, and the Sides C A, C B, equal, since the Triangle A C B is Equilateral: The Angles A C E, B C E, are equal, because the Angle A C B was Divided into two equal parts; therefore (*by the* 4th.) the Bases A E, E B, are equal.

USE.

USE.

THis Propoſition is often made uſe of, our ordinary Practice obligeth us thereto. Wherefore Geometricians would that a Line ſhould be divided at once, and not by ſeveral Trials, and that by an infallible method, the Practice of this is moſt uſeful in Dividing of Meaſures into ſmaller parts.

PROPOSITION XI.

PROBLEM.

FRom a Point C in a Right Line given A B, to erect a Right Line C F, at Right Angles.

Take on either ſide of this Point given two equal Lines, C D, C E, and upon the Right Line D E, erect an Equilateral Triangle; draw the Line F C, and it will be the Perpendicular required.

Demonſtration. The Triangles D C F, E C F, have the Side C F common, the Sides D F, E F, equal, the Baſes D C, E C, are alſo equal: Therefore (*by the* 8th.) the Angles D C F, E C F, are equal,

equal, *(and by the* 10th *Def.)* the Line C F is Perpendicular to A B.

PROPOSITION XII.

PROBLEM.

UPon an indeterminate Right Line B C, from a Point given, that is not in it as A, to let fall a Perpendicular A E.

From the Center A describe a Circle, cutting the Right Line given B C, in the Points G and F, then bisect G F in E, and draw the Right Line A E; which will be the Perpendidular required; draw the Lines A G, A F.

Demonstration. The Triangles G E A, F E A, have the Side A E common, and the Sides E G, E F, equal, the Line G F being bisected in E; the Bases A G, A F, being drawn from the Center A, to the Circumference F G, being therefore equal; thence the Angles A E G, A E F, are equal, *(by the* 8th*)* and the Line A E Perpendicular, *(by the* 10th *Def.)*

USE.

USE

THis Propoſition is not only uſed by Mathematicians, but almoſt by all Tradeſmen, as Joyners, Carpenters, Shipwrights, Maſons, and others: There Square being a Right Angle, in Fortifications after the French *manner, the Right Angled is preferred above all others. We cannot put Surveying in Practice, without making uſe of Perpendicular Lines: Dialling cannot be without it, in fine, the greateſt part of the practice of Tradeſmen, ſuppoſeth the knowledg of drawing Perpendicular Lines.*

PROPOSITION XIII.

THEOREM.

IF a Line fall on another Line, it ſhall make therewith Two Right Angles, or Two Angles equal to two Right Angles.

Let the Line A D fall upon B G: I ſay that it ſhall make therewith two Right Angles, or two Angles, the one Obtuſe, the other Acute; which being added togeher, ſhall be equal to two Right Angles.

Demonstration. Let the Line A D, be Perpendicular to B G, it is evident (*by the* 10th *Def.*) that the Angles A D B, A D G, are equal, and consequently are Right Angles.

Secondly, let the Line E D be drawn not at Right Angles to B G, draw the Perpendicular A D; (*by the* 11th) the Angles A D B, A D G, are both Right Angles, and are equal to the three Angles A D G, A D E, E D B. Now the Obtuse Angle E D G, and the Acute Angle E D B, are equal to the Three Angles A D G, A D E, E D B: Therefore the Angles E D G, E D B, are equal to Two Right Angles.

COROLLARIES.

1. HEnce if one Angle A B D be Right, the other A B G is also Right, if one Acute, the other Obtuse, and so on the contrary.

2. If the Line E D falling on B G, maketh the Angle E D B Acute, the Angle E D G shall be Obtuse.

USE.

USE.

WHen we know one of the Angles, which is made by a Right Line falling on another Right Line, we know also the other, as for instance, let E D B be 70 degrees, take away 70 degrees from 180, there remains E D G 110. The Practice of this is often of use in Trigonometry; and also in Astronomy, to find the excentricity of the Circle, which the Sun describeth in his Annual motion.

PROPOSITION XIV.
THEOREM.

IF Two Right Lines, meeting each other in the same Point in another Line, making therewith Angles equal to Two Right Angles, then shall those Lines make one streight Line.

Let the Lines C A, D A, meet in the same Point A, of the Line A B; and let the Adjacent Angles C A B, B A D, be equal to Two Right Angles. I say that then the Lines C A, A D, make one streight Line; for if C A be continued, it shall fall precisely on A D. If you

deny it, let CA, AE, make one Right Line, and describe a Circle from the Center A.

Demonstration. If you say CAE is a streight Line, the Arch CBE is a Semi-Circle: Now it is also supposed that the Angles CAB, BAD, are equal to Two Right Angles, that is to say, that their measure CBD is a Semi-Circle: Therefore the Arches CBE, CBD, shall be equal, which is impossible, since that the one is a part of the other; therefore the Line CA being continued, shall make one Line with AD.

PROPOSITION XV.
THEOREM.

IF Two Right Lines cut through one another, then are the Two Angles which are opposite equal one to the other.

Let the Lines AB, CD, cut each other in the Point E: I say that the Angles AEC, DEB, which are opposite are equal.

Demonstration. The Line CE falling on the Line AB maketh the Angles AEC, CEB, equal to Two Right Angles, *(by the 13th Prop.)* in the like manner doth the Line BE falling

falling on the Line CD, make the Angles CEB, BED, equal to Two Right Angles. Therefore the Angles AEC, CEB, taken together, are equal to the Angles CEB, BED, take away the Angle CEB, which is common to both, then *(by the 3d Axiom)* the Angles AEC, DEB, shall be equal.

Cor. If the Two Lines DE, EC, concurring in the same Point E, of the Line AB, making therewith the opposite Angles AEC, DEB, equal; DE, EC, shall be a streight Line.

Demonstration. The Line EC falling on AB, maketh the Angles AEC, BEC, equal to Two Right Angles *(by the 13th.)* It is also supposed that the Angle DEB is equal to the Angle AEC. Therefore the Angles DEB, BEC are equal to Two Right, and *(by the 14th.)* the Lines CE, ED, is one streight Line.

USE.

WE *often make use of the two Preceding Propositions, in* Catoptricks, *to prove that of all the Lines which can be drawn from the Point A, by reflection, to the Point B, those are the shortest which make the Angles of Incidence equal to the Angles of Reflection. For* Use 15.

Example,

Example, if the Angles *B E D*, *A E F*, are equal; the Lines *A E*, *E B*, are shorter then *A F*, *F B*. Draw from the Point *B*, the Perpendicular *B D*, and let the Lines *B D*, *D C*, be equal: Then draw *E C*, *F C*. Now in the first place, in the Triangles *B E D*, *C E D*, the Side *D E* being common, and the Sides *B D*, *D C*, being equal, and the Angles *C D E*, *B D E*, being also equal, the Bases *B E*, *C E*, shall be equal; as also the Angles *B E D*, *D E C*, (by the 4th) I might prove by the same manner of Argument, that *B F*, *C F*, are equal.

Demonstration. The Angles *B E D*, *D E C*, are equal, it is also supposed that the Angles *B E D*, *A E F*, are equal: Thence the opposite Angles *D E C*, *A E F*, shall be equal; and (by the Cor. of the 15th Prop.) *A E C* is a streight Line. By consequence *A F C* is a Triangle, in which the Sides *A F*, *F C*, are greater than *A E C*; that is to say *A E*, *E B*: For the Lines *A E*, *F C*, are equal to *A F*, *E B*: Therefore the Lines *A F*, *F B*, are greater then *A E*, *B E*; and since natural Causes do act by the most shortest Lines, therefore all reflections are made after this sort; that the

Angles

Angles of Reflection, and of Incidance, are equal.

Moreover, because we can easily prove that all the Angles which are made upon a Plain, by the meeting of never so many Lines in the same Point, are equal to Four Right Angles; since that in the first Figure of this Proposition the Angles A E C, A E D, are equal to two Right, as also B E D, B E C; we make this General Rule to determine the number of Polygons, which may be joyned together to Pave a Floor, so we say that four squares, six Triangles, three Hexigones, may Pave the same; and that it is for this reason the Bees make their cellules Hexagonal. Prop. XV.

PROPOSITION XVI.

THEOREM.

THe exteriour Angle of a Triangle is greater then either of the inward and Opposite Angles.

Continue the Side B C of the Triangle A B C: I say that the exteriour Angle A C D, is greater then the interiour opposite Angle A B C or B A C: Imagine that the Triangle A B C moveth along B D, and that it is Transported to C E D.

Demonstration.

Demonstration. It is impossible that the Triangle A B C should be thus moved, and that the Point A should not alter its place moving towards E. Now if it moveth towards E, the Angle E C D, that is to say A B C, is less than the Angle A C D; therefore the Interiour Angle A B C, is less than the Exteriour A C D.

It is easy to prove that the Angle A, is also less than the Exteriour A C D: For having continued the Side A C to F, the opposite Angles B C F, A C D, are equal (*by the* 15th.) and making the Triangle A B C to slide along the Line A C F, it is demonstrable, that the Angle B C F, is greater than the Angle A.

USE.

Use 16. *WE draw from this Proposition several very useful conclusions. The First, that from a given Point there cannot be drawn no more than one Perpendicular to the same Line. Example, Let the Line A B be Perpendicular to the Line B C: I say A C shall not be Perpendicular thereto, because the Right Angle A B C, shall be greater than the Interiour A C B:*

ACB: *Therefore ACB shall not be a Right Angle, neither AC a Perpendicular.*

Secondly, that from the same Point A, there can only be drawn two equal Lines; for Example, AC, AD, and that if you draw a Third AE, it shall not be equal. For since AC, AD, are equal, the Angles ACD, ADC, are equal, (by the 5th.) now in the Triangle AEC, the Exteriour Angle ACB is greater than the Interiour AEC; and thus is the Angle ADE greater than AED. Therefore the Line AE, greater than AD; and by consequence AC, AE, are not equal.

Thirdly, if the Line AC maketh the Angle ACB acute, and ACF obtuse, the Perpendicular drawn from A, shall fall on the same Side which the Acute Angle is of; for if one should say that AE is a Perpendicular, and the Angle AEF is Right; then the Right Angle AEF would be greater than the Obtuse Angle ACE. Those conclusions we make use of to measure all Parallelograms, Triangles, and Trapeziams, and to Reduce them into Rectangular Figures.

PROP.

PROPOSITION XVII.

THEOREM.

TWo Angles of any Triangle, are leſs than Two Right Angles.

Let A B C be the propoſed Triangle; I ſay that Two of the Angles taken together B A C, B C A, are leſs than Two Right Angles. Continue the Side C A, to D.

Demonſtration. The Interiour Angle C, is leſs than the Exteriour B A D, (*by the* 16th.) Add to both the Angle B A C; the Angles B A C, B C A, ſhall be leſs than the Angles B A C, B A D; which latter are equal to Two Right (*by the* 13th.) Therefore the Angles B A C, B C A, are leſs than Two Right.

I might Demonſtrate after the ſame manner, that the Angles A B C, A C B, are leſs than two Right, by continuing the Side B C.

Coroll. If one Angle of a Triangle be either Right or Obtuſe, the other Two ſhall be Acute.

This Propoſition is neceſſary to Demonſtrate thoſe which follow.

PROP.

PROPOSITION XVIII.
THEOREM.

THe greateſt Side of every Triangle, ſubtends the greateſt Angle.

Let the Side B C, of the Triangle A B C, be greater then the Side A C. I ſay that the Angle B A C, which is oppoſite to the Side B C, is greater than the Angle B, which is oppoſite to the Side A C. Cut off from B C, the Line C D, equal to A C and draw A D.

Demonſtration. Seeing that the Sides A C, C D, are equal, the Triangle A C B is an Iſoſceles Triangle, (*by the* 5th.) the Angles C D A, C A D, are therefore equal. Now the whole Angle B A C is greater than the Angle C A D: Thence the Angle B A C, is greater than the Angle C D A; which being exteriour, in regard of the Triangle A B D, is greater than the interiour B (*by the* 16th.) Therefore the Angle B A C, is greater than the Angle B.

PRO-

PROPOSITION XIX.

THEOREM.

IN every Triangle, the greateſt Angle is oppoſite to the greateſt Side.

Let the Angle A of the Triangle BAC, be greater than the Angle ABC. I ſay that the Side BC, oppoſite to the Angle A, is greater then the Side AC, oppoſite to the Angle B.

Demonſtration. If the Side BC, was equal to the Side AC; in this caſe the Angles A and B would be equal (*by the* 5th.) which is contrary to the Hypotheſis: If the Side BC was leſs than AC, then the Angle B would be greater than A, which is alſo contrary to the Hypotheſis. Wherefore I conclude that the Side BC is greater than AC.

USE.

Uſe 19. *WE prove by theſe Propoſitions, not only that from the ſame Point to a Line given, there can be but one Perpendicular drawn, but alſo that that Perpendicular is the ſhorteſt Line of all thoſe Lines which might be drawn to the ſaid*

Line. As for inſtance, if the Line RV be Perpendicular to ST; it ſhall be ſhorter then RS: becauſe the Angle RVS being Right, the Angle RSV ſhall be Acute (by the Cor. of the 17th) *and the Line RV ſhall be ſhorter than RS* (by the Preceding.) *For this Reaſon Geometricians always make uſe of the Perpendiculars in their meaſuring; and Reducing irregular Figures into thoſe whoſe Angles are Right.*

I further add, that ſeeing there can only be drawn Three Perpendiculars to one and the ſame Point, it cannot be imagined that there are more than Three Species of quantity; viz. *A Line, a Surface, and a Solid.*

We alſo prove by theſe Propoſitions, that a Boul which is exactly round, being put on a Plain, cannot ſtand but on one determinate Point. As for Example, Let the Line AB Repreſent the Plain, and from the Center of the Earth C, let the Line CA be drawn Perpendicular to the Line AB. I ſay that a Boul being placed on the Point B, ought not to ſtand on that Point, becauſe no heavy Body will ſtand ſtill while it may Deſcend. Now the Boul B going towards A, is always Deſcending, and cometh nearer and nearer

the

the Center of the Earth C: Because in the Triangle CAB, the Perpendicular CA is shorter than BC.

We also prove in like manner, that a liquid Body ought to Descend from B to A, and that the surface ought to be round.

PROPOSITION XX.

THEOREM.

THe Two Sides of a Triangle taken together are greater then the Third.

I say that the Two Sides TL, LV, of the Triangle TLV, are greater than the Side TV. Some prove this Proposition, by the Definition of a streight Line, which is the shortest which can be drawn between Two Points: Therefore the Line TV is less than the Two Lines TL, LV.

But we may Demonstrate the same another way, continue the Side LV to R, and let LR, LT, be equal, then draw the Line RT.

Demonstration. The Sides LT, LR, of the Triangle LTR, are equal. Therefore the Angles R and RTL equal (*by the* 5 h.) but the Angle RTV, is greater than the Angle RTL: Therefore

fore the Angle R T V is greater than the Angle R, and (*by the* 17th) in the Triangle R T V, the Side R V, that is to ſay the ſum of the Sides L T, L V, are greater than the Side T V.

PROPOSITION XXI.

THEOREM.

IF on the ſame Baſe you draw a leſſer Triangle in a greater, the Sides of the leſſer ſhall be ſhorter than the greater, but contain a greater Angle.

Let the leſs Triangle A D B, be drawn within the greater A C B, on the ſame Baſe A B. I ſay in the firſt place that the Sides A C, B C, are greater than the Sides A D, B D. Continue the Side A D unto E.

Demonſtration. In the Triangle A C E the Sides, A C, C E, taken together, are greater than the Side A E, (*by the* 20th.) Add thereto the Side E B, the Sides A C, C E B, ſhall be greater than the Sides A E, E B. Likewiſe in the Triangle D B E, the Two Sides B E, E D, taken together, are greater than B D; and adding thereto A D, the Sides A E, E B, ſhall be greater than A D, B D. More-

Moreover, I say that the Angle ADB, is greater than the Angle ACB: For the Angle ADB is exteriour in respect of the Triangle DEB; it is therefore greater than the Interiour DEB *(by the 16th.)* Likewise the Angle DEB being Exteriour in respect of the Triangle ACE, is greater than the Angle ACE: Therefore the Angle ADB, is greater than the Angle ACB.

USE

Prop. XXI. *WE Demonstrate in Opticks by this Proposition, that if from the Point C, one should see the Base AB, it would seem less than if one should see the same from the Point D; according to this Principle, that quantities seen under a greater Angle, appear greater, for which reason* Vitruvius *would that the Tops of very high Pillars should be made but little tapering, because that their Tops being at a good distance from the Eyes, will of themselves appear very much diminished.*

PRO-

PROPOSITION XXII.

PROBLEM.

TO make a Triangle having its Sides equal to Three Right Lines given, provided that Two of them be greater than the Third.

Let it be proposed to make a Triangle, having its Three Sides equal to the Three given Lines, A B, D, E, take with your Compasses the Line D, and putting one Foot thereof in the Point B, make an Arch: Then take in your Compasses the Line E, and putting one Foot in the Point A, cross with the other Foot the former Arch in C; Draw the Lines A C, B C. I say that the Triangle A B C, is what you desire.

Demonstration. The Side A C is equal to the Line E, since it is the Radius of an Arch drawn on the Center A, equal in Length to the Line E, likewise the Side B C is equal to the Line D: Therefore the Three Sides A C, B C, A D, are equal to the Lines E, D, A B.

USE.

USE.

WE make use of this Proposition to make a Figure equal, or like unto another, for having Divided the Figure into Triangles; and making other Triangles, having their Sides equal to the sides of the Triangles in the proposed figure, we shall have a Figure like unto the same in all Respects. But if we desire it should be only like thereunto, but lesser, for Example, if we would have the Form of any Plain or Piece of Ground on Paper; having Divided the same into Triangles, and measured all their Sides, we make Triangles like unto those of the Plain, by the help of a Scale of equal parts, from which we take the number of Parts, which their Sides contain, whether Feet, Rods, or any other measure, and applying them as is here Taught.

PRO-

PROPOSITION XXIII.
PROBLEM.

TO make an Angle equal to an Angle given in any Point of a Line.

Let it be propoſed to make an Angle equal to E D F, at the Point A, of the Line A B, at the Points A and D, as Centers, draw two Arches B C, E F, with the ſame extent of the Compaſſes; then take the Diſtance E F between your Compaſſes, put one Foot in B, and cut off B C, and draw A C. I ſay that the Angles B A C, E D F, are equal.

Demonſtration. The Triangles A B C, D E F, have the Sides A B, A C equal to the Sides D E, D F; ſince that the Arches B C, E F, were deſcribed with the ſame extent of the Compaſs, they have alſo their Baſes B C, E F, equal: Therefore the Angles B A C, D E F, are equal (*by the* 8th.)

USE.

THis Problem is ſo neceſſary in Surveying Fortifications, Proſpective, Dialling, and in all other parts of the Mathematicks; ſo that the greateſt part

of their Practice would be impossible, if one Angle could not be made equal to another, or of any number of Degrees required.

PROPOSITION XXIV.

THEOREM.

IF Two Triangles which have Two Sides of the one equal to the Two Sides of the other; that which hath the greatest Angle hath the greatest Base.

Let the Sides A B, D E, A C, D F, of the Triangles A B C, D E F, be equal; and let the Angle B A C, be greater than the Angle E D F. I say that the Base B C, is greater than the Base E F: Make the Angle E D G, equal to the Angle B A C, (*by the* 23^d.) and the Line D G equal to A C, then draw E G. In the first place the Triangles A B C, D E G, having the Sides A B, D E, A C, D G, equal; and the Angle E D G, equal to B A C; their Bases B C, E G, are equal (*by the* 4th.) and the Lines D G, D F, being equal to A C, they shall be equal amongst themselves.

Demonstration.

Demonſtration. In the Triangle D G F, the Sides D G, D F, being equal, the Angles D G F, D F G, are equal (*by the* 5th.) but the Angle E G F, is leſs than D G F, and the Angle E F G, is greater than D F G. Therefore in the Triangle E F G, the Angle E F G, ſhall be greater than the Angle E G F, thence (by the 18th.) the Line E G, oppoſite to the greateſt Angle E F G, ſhall be greater than E F; thence B C, equal to E G, is greater than the Baſe E F.

PROPOSITION XXV.

THEOREM.

OF Two Triangles which have Two Sides of the one equal to Two Sides of the other, that Triangle which hath the greateſt Baſe hath alſo the greateſt Angle.

Let the Sides A B, D E, A C, D F; of the Triangles A B C, D E F, be equal; and let the Baſe B C be greater than the Baſe E F. I ſay that the Angle A ſhall be greater than the Angle D.

Demonſtration. If the Angle A were not greater than the Angle D, it would be equal, or leſs; if equal, in

this case the Bases B C would be equal (*by the* 4th.) If less, then the Base E F would be greater than the Base B C, (*by the* 24th.) both which is contrary to our Hyp.

These Propositions are of use to Demonstrate those which follow.

PROPOSITION XXVI.

THEOREM.

A Triangle which hath One Side and Two Angles equal to those of another, shall be equal thereto in every respect.

Let the Angles A B C, D E F; A C D, D F E; of the Triangles ABC, DEF, be equal; and let the Sides B C, E F, which are between those Angles, be also equal to each other. I say that their other Sides are equal: For Example, A C, D F; but let it be imagined that the Side D F is greater than A C, and that G F is equal to A C; and draw the Line G E.

Demonstration. The Triangles A B C, G E F, have the Sides E F, B C, A C, G F, equal; the Angle C is also supposed equal to the Angle F, thence (*by the* 4th.) the Triangles A B C, G E F,

G E F, are equal in every reſpect, and the Angles G E F, A B C, are equal; but according to our firſt Hyp. the Angles A B C, D E F, were equal; by this Argument the Angles D E F, G E F, would be equal; that is to ſay the whole equal to a part, which is impoſſible; therefore D F cannot be greater than A C, nor A C greater than D F; becauſe the ſame Demonſtration might be made in the Triangle A B C.

Secondly, Let us ſuppoſe that the Angles A and D, C and F, are equal; and that the Sides BC, EF, which are oppoſite to the equal Angle A and D, are alſo equal to each other. I ſay the other Sides are equal; for if DF be greater than A C, make GF equal to A C, and draw the Line G E.

Demonſtration. The Triangles ABC, G E F, have the Sides E F, B C, F G, C A, equal, they are then equal in every reſpect (*by the* 4th.) and the Angles E G F, B A C, ſhall be equal, but according to our Hyp. A and D were equal, thus the Angles D, and E G F, ſhould be equal, which is impoſſible, ſince that the Angle E G F being exteriour, in reſpect of the Triangle E G D, it ought to be greater than the

Interiour Angle D, (*by the* 16.h) Therefore the Side DF, is not greater than A C.

USE.

Use 26. THales *made use of this Proposition to measure inaccessible Distances, the Distance A D being proposed from the Point A, he draws the Line AC Perpendicular to A D; then placing a Semicircle at the Point C, he measureth the Angle A C D, then he takes an Angle equal thereto on the other side, drawing the Line C B, untill it meets the Line D A, continued to the Point B. He Demonstrates that the Lines A D, A B, were equal; so measuring actually the accessible Line, he might know by that means the other: For the Two Triangles A D C, A B C, have the Right Angles C A D, C A B, equal; both the Angles A C D, A C B, were taken equal to each other; and the Side A C is common to both Triangles. Therefore* (by the 26.h) *the Sides A D, A B, are equal.*

LEM-

LEMMA.

A Line which is Perpendicular to one Parallel, is alſo Perpendicular to the other. Lem. 26.

Let the Lines AB, CD, be Parallel to each other, and let EF be Perpendicular to CD. I ſay that it is Perpendicular to AB. Cut off two equal Lines CF, FD; at the Points C and D erect two Perpendiculars to the Line CD, which ſhall alſo be equal to FE, by the Definition of Parallels; and draw the Lines EC, ED.

Demonſtration. The Triangles CEF, FED, have the Side FE common; the Sides FD, FC, are equal; the Angles at F are Right, and by conſequence equal. Therefore (by the 4th) *the Baſes EC, ED, the Angles FED, FEC; FDE, FCE, are equal; and thoſe two laſt being taken away from the Right Angles ACE, BDF, leaveth the equal Angles EDB, ECA: Now the Triangles CAE, DBE, ſhall* (by the 4th.) *have the Angles DEB, CEA, equal; which Angles being added to the equal Angles CEF, FED, maketh equal Angles*

F E B, F E A. Therefore E F is Perpendicular to A B.

PROPOSITION XXVII. THEOREM.

IF a Right Line falling upon Two Right Lines, make the Alternate Angles equal, the one to the other, then are the Right Lines Parallel.

Let the Line E H fall on the Right Lines A B, C D, making therewith, the Alternate Angles A F G, F G D, equal. I say in the first place, that the Lines AB, CD, shall not meet, although continued as far as one lists. For supposing they should meet in I, and that F B I, C D I, are two streight Lines.

Demonstration. If F B I, G D I, be two streight Lines; then F I G is a Triangle; then (*by the* 16th.) the exteriour Angle A F G shall be greater than the interiour F G I. Wherefore that the equality of the Angles may subsist; the Lines A B, C D, must never meet each other.

But because we have Examples of some crooked Lines that never intersect, which notwithstanding are not Parallels, but approach continually.

To

To prove the foregoing, I make another Demonstration, as followeth, First,

I say that if the Line E H falleth on the Lines A B, C D, maketh the Alternate Angles A F G, F G D, equal; the Lines A B, C D, are Parallels, that is in every part equidistant from each other; for which reason the Perpendiculars shall be equal to each other. Draw from G, to the Line A B, the Perpendicular G A; and C D being taken equal to A F, draw F D.

Demonstration. The Triangles A G F, F G D, have the Side G F common; the Side G D, was taken equal to A F, it is supposed that the Angles A F G, F G D, are equal. Therefore *(by the* 4th.) A C, F D, are equal, and the Angle G D F is equal to the Right Angle G A F. Thence F D is Perpendicular. Furthermore that A B is Parallel to C D; for the Parallel to C D, is to be drawn from the Point F, and must pass through the Point A, according to our Definition of Parallels, which is that the Perpendiculars A G, F D, are equal.

PROPOSITION XXVIII.

THEOREM.

IF a Right Line falling upon Two Right Lines, make the Exteriour Angle equal to the Interiour oppoſite Angle of the other on the ſame Side, or the Two Interiour on the ſame Side equal to Two Right, then are the Right Lines Parallel.

Having drawn a Figure like unto the former, let the Line EH fall on AB, CD, make in the firſt place the Exteriour Angle EFB, equal to the Interiour oppoſite Angle of the other on the ſame Side FGD. I ſay that the Lines AB, CD, are Parallel.

Demonſtration. The Angle EFB, is equal to the Angle AFG, (*by the* 15th.) and it being ſuppoſed that EFB, is alſo equal to FGD Thence the Alternate Angle AFG, FGD, ſhall be equal: and (*by the* 27th.) the Lines AB, CD, are Parallel.

Secondly, I ſay that if the Angles BFG, FGD, which are Interiour on the ſame Side, be found equal to Two Right Angles. the Lines AB, CD, ſhall be Parallel.

Demonſtration.

Demonſtration. The Angles A F G, B F G, are equal to two Right (*by the* 13th.) and it is ſuppoſed that the Angles G F B, F G D, are equal to Two Right; therefore the Angles A F G, F G D, are equal: And (*by the* 27th) the Lines A B, C D, ſhall be Parallel.

PROPOSITION XXIX.

THEOREM.

IF a Right Line fall upon Two Parallels, the Alternate Angles ſhall be equal: The Exteriour Angle ſhall be equal to the Interiour and Oppoſite; and the Two Interiour Angles on the ſame Side ſhall be equal to Two Right Angles.

Let the right Line E H fall on two Parallels AB, CD. I ſay in the firſt place that the Alternate Angles A F G, F G D, are equal. Draw from the Points F and G the Perpendiculars G A, F D, which by the Definition of Parallels are equal.

Demonſtra. In the rectangled Triangles AFG, FGD, the Sides FD, AG, being equal, as alſo the Right Angles A and D; and the Side F G being common: I ſay in the firſt place that the Side G D is equal to A F. For if G D were

were greater; then having cut off the Line D I equal to A F, and draw F I; then the Triangles A F G, F D I, would have their Bases G F, F I, equal, which cannot be; since that the Angle D being Right, the Angle F I D is Acute, and F I G Obtuse (*by the* 13th.) and (*by the* 18th,) in the Triangle F I G, the Side F G opposite to the Obtuse Angle F I G, is greater than F I. Wherefore D G equal to A F; and the Triangles A F G, F G D, having all the Sides equal, the Alternate Angles A F G, F G D, equal, because opposite to the equal Sides A G, F D.

Again, I say that the Exteriour Angle E F B is equal to the Interiour F G D; because (*by the* 15th,) it is equal to its opposite A F G, which is equal to his Alternate F G D.

Now, Since the Angles A F G and G F B are equal to Two Right; take away A F G, and in the place thereof substitute his Alternate F G D, the Interiour Angles G F B, F G D, shall be equal to Two Right Angles.

USE.

USE.

ERatosthenes *applyeth those Propositions to the measuring of the Circumference of the Earth. He supposeth that two Rays of Light proceeding from the Center of the Sun to Two Points on the Earth, are Physically Parallel: He supposeth also that at* Syena *a Town in* Egypt, *the Sun was in the zenith on the day of the Solstice, because that their Cisterns, Wells, and such like, were enlightned by the Suns Rays to the very bottom, he measured also the distance from* Alexandria *to* Syena. *To Demonstrate which, let us suppose* Alexandrea *to be Situated at the Point B, and* Syena *at the Point A, where let the Style BC be erected Perpendicular to the Horizon, and let DF, EC, be Two Rays proceeding from the Center of the Sun, when in the Solstice, which are Parallels between themselves; let DA that Ray of Light Perpendicular to* Syena, *be supposed to be continued to F, the Center of the Earth.* Use 29.

Now having observed at Alexandria *the Angle GCB, which is made or included between the Perpendicular CB, and the Ray of Light EC: I say that because*

becauſe the aforeſaid Rays are Parallel, the Alternate Angles G C B, B F A, are equal. We know then the Angle A F B, and the meaſure of A B the diſtance of Alexandria *to* Syena *taken in Degrees. Now ſuppoſing this Diſtance be known in Miles, the Circumference of the Earth ſhall be eaſily found by a ſimple Rule of Three:* For if ſo many Degrees give ſo many Miles, what ſhall 360.

PROPOSITION XXX.

THEOREM.

Right Lines Parallel to one and the ſame, are alſo Parallel the one to the other.

Let the Lines A B, F E, be Parallel to the Line C D; I ſay they are Parallel the one to the other. Let the Line G K cut all three.

Demonſtration. Seeing that the Lines A B, C D, are Parallel; the Alternate Angles A H I, H I D, ſhall be equal (*by the* 29^{h}:) and becauſe the Lines C D, F E, are alſo Parallel, the Exteriour Angle H I D ſhall be equal to the Interiour I L E (*by the ſame.*) Therefore the Alternate Angles A H I, I L E, ſhall

ſhall be equal, and the Line A B, F E, Parallels (*by the* 28th.)

PROPOSITION XXXI.

PROBLEM.

TO Draw a Line Parallel to another, through a Point given.

It is propoſed to draw a Line through the Point C, which ſhall be Parallel to the Line A B. Draw the Line C E, and make the Angle E C D, equal to the Angle C E A: I ſay that the Line C D is Parallel to A B.

Demonſtration. The Alternate Angles D C E, C E A, are equal: Therefore the Lines C D, A B, are Parallel.

One might eaſily demonſtrate the Eleventh Maxim, that is to ſay, that if a Line falling on Two other Lines, maketh the Interiour Angles leſs than Two Right, thoſe Two Lines ſhall Interſect.

Let the Line A C falling on the Lines A B, C D, make the Interiour Angles A C D, C A B, leſs than two Right: I ſay the Lines A B, C D, ſhall Interſect. Let the Angles A C D, C A E, be equal to Two Right, the Lines A E, C D, are Parallels Ax. 11.

(by

(by the 29th.) Take at discretion the Line *A B*, and through the Point *B* draw the Line *E F* Parallel to *C A*. Then take *E B* as many times as shall be necessary to fall below the Line *C D*, as in this Figure I have taken it but twice, wherefore *E B*, *B F*, are equal. Draw from the Point *F* a Parallel *F G*, equal to *A E*, and joyn *G B*. I say that the Line *A B G*, is but one streight Line; and that so the Line *A B* concurring with *F G*, if *C D* be continued, since it cannot cut the Parallel *F G*, it shall cut the Line *B G* between *B* and *G*.

Demonstration. The Triangles *A E B*, *E F G*, have the Sides *A E*, *F G*; *B E*, *B F*, equal, as also the Alternate Angles *A E B*, *B F G*, (by the 29th.) Therefore they are equal in every respect (by the 4th.) and the opposite Angles *A B E*, *F B G*, are equal; and consequently (according to the 15th.) *A B*, *B G*, maketh one streight Line.

USE

THe use of Parallel Lines is very necessary: First in perspective, since that the Appearances or Images of the Parallel Lines on the Draft are Parallels between

between themſelves. In Navigation, like Rumbs in our Charts, are Parallel to each other. Likewiſe the hour Lines on Polar Dials, the Compaſs of Proportion or Sector is grounded alſo on Parallel Lines.

PROPOSITION XXXII.

THEOREM.

THe Exteriour Angle of a Triangle is equal to the Two Interiour oppoſite Angles taken together, and the Three Angles of a Triangle are equal to Two Right.

Let the Side BC of the Triangle ABC, be continued to D. I ſay that the Exteriour Angle ACD, is equal to the Two Interiour Angles A, and B, taken together. Draw from the Point C the Line CE, Parallel to AB.

Demonſtration. The Lines AB, CE, are Parallels; therefore *(by the* 29th,) the Alternate Angles ECA, CAB, are equal; and *(by the ſame)* the Exteriour Angle ECD is equal to the Interiour B. By conſequence the whole Angle ACD, is equal to the Two Angles ACE, ECD, of whom it is compoſed, it ſhall then be equal to A and B taken together.

In

In the Second place, the Angles A C D, A C B, are equal to Two Right (*by the* 13*th.*) And I have Demonstrated that the Angle A C D was equal to the Angles A and B, taken together: Therefore the Angle A C D, is equal to A and B, that is to say all the Angles of the Triangle A B C, are equal to Two Right, or 180 degrees.

Corollary. 1. The Three Angles of one Triangle, are equal to the Three Angles of another Triangle.

Coroll. 2. If Two Angles of a Triangle be equal to Two Angles of another Triangle; the remaining Angle in the one shall be equal to the remaining Angle in the other.

Coroll. 3. If a Triangle have one of its Angles Right, the other Two shall be Acute; and being taken together, shall be equal to One Right Angle.

Coroll. 4. From one and the same Point of a Line there can be drawn but one Perpendicular; because a Triangle cannot have Two Right Angles.

Coroll. 5. The Perpendicular is the shortest Line which can be drawn from a Point to a Line.

Coroll. 6.

Coroll. 6. In a Right Angled Triangle, the greateſt Angle is a Right Angle, and the longeſt Side is oppoſite thereto.

Coroll. 7. Each Angle of an Equilateral Triangle containeth 60 degrees, that is to ſay the third of 180.

USE.

THis Propoſition is of uſe to us in Aſtronomy, to determine the Parallax. Let the Point A repreſent the Center of the Earth; and from the Point B on the Superficies of the Earth, let there be taken by obſervation the Angle D B C, that is to ſay the apparent Diſtance of a Plannet or Commet from the Zenith D. I ſay if the Earth were tranſparent, this Planet or Commet viewed from the Center of the Earth A, would appear diſtant from the Zenith D, equal to the quantity of the Angle C A D, which is leſs than the Angle C B D. For the Angle C B D being Exteriour in reſpect of the Triangle A B C, is equal (by the 32d.) *to the oppoſite Angles A and C. Whence the Angle C ſhall be equal to the exceſs of the Angle C B D, above the Angle A. From whence I conclude, that if I know by an Aſtronomical* Uſe 32.

mical Table, how far that Planet or Comnet ought to appear Distant from the Zenith to one which should be in the Center of the Earth, and if I observe at the same time, the difference of those Two Angles shall be the Parallax, viz. *The Angle B C A.*

PROPOSITION XXXIII.

THEOREM.

IF Two equal and Parallel Lines be joyned together with Two other Right Lines, then are those Lines also equal and Parallel.

Let the Lines A B, G D, be Parallel and equal, and let them be joyned with A G. B D. I say that the Lines AG, B D, are equal and Parallel, draw the Diagonal B G.

Demonstration. Seeing the Lines AB, G D, are parallel, the Alternate Angles A B G, B G D, shall be equal (*by the* 29th.) and the Side G B being common to both the Triangles A B G, B G D, and the Sides AB, G D, equal, with the Angles A B G, B G D, equal, as before, the Bases of those Triangles A G, B D, shall be equal (*by the* 4.) as also the Angles D B G, B G A; which

which because Alternate, the Lines AG, BD, shall be Parallel (*by the* 27th.)

USE.

THis Proposition is put in practice to measure as well the Perpendicular height A G of Mountains as their Horizontal Lines C, G, which are hidden in their thickness. To effect which, we make use of a very large square A B D, putting one end thereof in A, in such manner that the other Side thereof B D may be Perpendicular to the Horizon, then we measure the Sides A D, B D, then we do the like again at the Point B, and measure B E, E C, the Sides Parallel to the Horizon, that is to say A D, B E, being added together, gives the Horizontal Line C G, and the Perpendicular Sides B D, E C, being added gives the Perpendicular Height A G. Use 33.

PRO-

PROPOSITION XXXIV.
THEOREM.

THe Sides, and the oppoſite Angle in a Parallelogram are equal, and the Diamter doth divide the ſame into Two equal parts.

Let the Figure AB, CD, be a Parallelogram, that is to ſay, let the Sides AB, CD; AC, BD, be Parallel. I ſay that the oppoſite Sides AB, CD; AC, BD, are equal; as alſo the Angles A and D; ABD, ACD; and that the Diameter BC doth divide the Figure into Two equal parts.

Demonſtration. The Lines AB, CD, are ſuppoſed Parallel: Therefore the Alternate Angles ABC, BCD, ſhall be equal (*by the* 29th.) Likewiſe the Sides AC, BD, being ſuppoſed Parallel; the Alternate Angles ACB, CBD, are equal. Both which Triangles have the ſame Side BC, and the Angles ABC, BCD; ACB, CBD, equal, they ſhall be equal in every reſpect, (*by the* 26th.) Therefore the Sides AB, CD; AC, BD; and the Angles A and D are equal; and the Diameter

ameter divides the Figure into Two equal parts; and ſeeing the Angles ABC, BCD; ACD, CBD, are equal, adding together ABC, CBD; BCD, ACD, we conclude that the oppoſite Angles ABD, ACD, ſhall be equal.

USE

Surveyers have ſometimes occaſion to make uſe of this Propoſition, to part or divide Lands. If the Field be a Parallelogram, it may be divided into Two equal parts by the Diameter AD. But if one be obliged to divide the ſame into Two equal parts from the Point E; divide the Diameter into two equal parts in the Point F, and draw the Line EFG, that Line ſhall divide the Field into Two equal parts; for the Triangle AEF, GFD, which have their Alternate Angles EAF, FDG; AEF, FGD; and the Sides AF, FD, equal; are equal (by the 26.ᵗʰ.) *And ſince the Trapezium BEFD, together with the Triangle AFE, that is to ſay, the Triangle ADB, is one half of the Parallelogram* (by the 34.ᵗʰ.) *the ſame Trapezium EFBD, together with the Triangle DFG, ſhall be one half of the Figure* Uſe 34.

Figure or Field, and the Line E G divideth the same into Two equal parts.

PROPOSITION XXXV.

THe Parallelograms are equal, when having the same Base, they are between the same Parallel Lines.

Let the Parallelograms A B E C, A B D F, have the same Base A B, and be between the same Parallels A B, C D: I say they are equal.

Demonstration. The Sides A B, C E, are equal (*by the* 34th.) as also A B, F D; wherefore C E, F D, are equal; and adding E F to each, the Lines C F, E D, shall be equal; the Triangles C F A, E B D, have the Sides C A, E B; C F, E D, equal with the Angles D E B, F C A, (*by the* 29th.) the one being Exteriour, and the other Interiour on the same Side, whence (*by the* 4th.) the Triangles A C E, B E D, are equal; and taking away from both that which is common to both, that is to say the little Triangle E G F; the Trapezium F G B D, shall be equal to the Trapezium C A G E; and adding to both the

the little Triangle A G B, the Parallelograms A B E C, A B D F, shall be equal.

THEOREM

Demonstration by the method of Indivisibles.

THis method is new invented by Cavalerius, *which is approved of by some, but rejected by others. It consisteth in this imagination or supposition, that Superficies are Composed of Lines like so many threads. Now it is certain that two pieces of Cloth shall be equal, if in both there be found the same number of Threads equal in length, and that they are as close joyned or woven in the one, as in the other.*

Let there be proposed two Parallelograms A B E C, A B D F, on the same Base A B, and between the same Parallels A B, C D. Moreover draw in the Parallelogram A B E C, as many Lines as you please, which let be continued into the other Parallelogram, it is evident that there will be no more Lines in the one than in the other; and that they shall be equal in length, that is to say to the Base A B; neither shall they be more close joyned in the one, than in the other; wherefore the Parallelograms are equal. Fig. 35.

PROPOSITION XXXVI.

THEOREM.

THe Parallelograms are equal, which being between the same Parallels have equal Bases.

Let the Bases CB, OD, of the Parallelograms ACBF, ODEG, be equal, and be both between the Parallels AE, CD. I say that those Parallelograms are equal; draw the Lines CG, BE.

Demonstration. The Bases CB, OD, are equal: OD, GE, are also equal: Whence CB, GE, are equal and Parallel; and by consequence (*according to the* 33^d.) CG, BE, shall be equal and Parallel; and CBEG shall be a Parallelogram equal to CBFA (*by the* 35th.) because they have the same Bases. In like manner taking GE for Base, the Parallelograms GODE, CBEG, are equal (*by the same.*) Wherefore the Parallelograms ACBF, ODEG, are equal.

USE.

USE of the foregoing.

WE Reduce Oblique Angled Parallelograms, as CBEG, or ODEG, into Rectangled Parallelograms; as CBFA, whereby, by measuring this laſt, which is eaſily done thus; Multiply AC, by CB, the Product ſhall be equal to the Parallelogram ACBF, and conſequently to the Parallelograms CBEG, or ODEG. Uſe 36.

PROPOSITION XXXVII.

THEOREM.

THe Triangles are equal, which having the ſame Baſe, are between the ſame Parallels.

The Triangles ACD, CDE, ſhall be equal, if they have the ſame Baſe CD, and if they are included between the Parallels AF, CH. Draw the Lines DB, DF, Parallel to the Lines AC, CE, and you will have made two Parallelograms.

Demonſtration. The Parallelograms ACDB, ECDF, are equal, (*by the* 36*th*.) The Triangles ACD, CDE,

are equal to the one half of those Parallelograms (*by the* 34th.) Therefore the Triangles A C D, C D E, are equal.

PROPOSITION XXXVIII.

THEOREM.

THe Triangles are equal, which having equal Bases, are between the same Parallels.

The Triangles A C D, G E H, are equal if they have their Bases C D, G H, equal, and if they be included between the same Parallels A F, C H. Draw the Lines B D, H F, Parallel to the Sides A C, E G; and you shall have made two Parallelograms.

Demonstration. The Parallelograms A C D B, E G H F; are equal *by the* 36th.) the Triangles A C D, E G H, are the one half of those Parallelograms (*by the* 34th.) Thence by consequence they are equal to each other.

USE.

Use 38. *IN the two last Propositions we have the practical way of Dividing a Triangular*

angular Field into two equal parts; for example A B C. Divide the Line B C, which let be taken for the Baſe, into Two equal parts in D: I ſay the Triangles A B D, A D C, are equal. For if you imagine a Line Parallel to B C, paſſing by the Point A, thoſe Triangles ſhall have equal Baſes, and ſhall be between the ſame Parallels, and by conſequence equal. We might make other Diviſions grounded on the ſame Propoſition, which I omit for brevity ſake.

PROPOSITION XXXIX.

PROBLEM.

EQual Triangles ſtanding on the ſame Baſe, are between the ſame Parallels.

If the Triangles A B C, D B C, which have the ſame Baſe B C, are equal; the Line A D drawn through the tops of thoſe Triangles, ſhall be Parallel to the Baſe. For if A D, B C, are not Parallels; having drawn a Parallel through the Point A, the ſame ſhall fall either under A D, as A O; or above as A E. Let us ſuppoſe it falleth above, continue B D, untill it meets A E in the Point E; then draw the Line C E.

Demonstration. The Triangles ABC, EBC, are equal (*by the* 38th.) seeing the Lines AE, BC, are Parallel, it is also supposed that the Triangles ABC, DBC, are equal; thence the Triangles DBC, EBC, should be equal; which is impossible, the first being a part of the second. From whence I conclude that the Parallel to BC cannot be drawn above as AE.

I further say, that it cannot fall under AD, as AO; because then the Triangle BOC should be equal to ABC, by consequence to the Triangle DBC, that is to say a part equal to the whole. It must then be granted that the Line AD is Parallel to BC.

PROPOSITION XL.

THEOREM.

EQual Triangles standing upon equal Bases, taken on the same Line, are betwixt the same Parallels.

If the equal Triangles ABC, DEF, have their Bases AB, DE, equal, and taken on the same Line AE, the Line CF drawn through the Vertices of those Triangles, shall be Parallel to AE;

AE; for if it be not Parallel, having drawn from the Point C a Parallel Line to AE, that Line ſhall fall above CF, as CG, or underneath as CI.

Demonſtration. If it paſſeth above CF, as CG, continue DF untill it meet CG in G, and draw EG; the Triangles ABC, DGE, would then be equal (*by the* 38th.) and having ſuppoſed that ABC, DEF, are equal, DFE, DGE, ſhould be alſo equal; which cannot be true, the one being a part of the other; therefore the Parallel paſſeth not above CF. I further add, that it cannot paſs underneath as CI; becauſe the Triangles ABC, DEI, would be equal to each other, and by conſequence DEI, DEF, the part and the whole would be equal, which is abſurd. Therefore CF is the only Line which is Parallel to AE.

PROPOSITION XLI.

THEOREM.

IF a Parallelogram have the ſame Baſe with a Triangle, and be between the ſame Parallels, then is the Parallelogram double to the Triangle.

If the Parallelagram ABCD, and

the Triangle E B C have the same Base B C, and between the same Parallels A E, B C; the Parallelogram shall be double to the Triangle, draw the Line A C.

Demonstration. The Triangles A B C, B C E, are equal (*by the* 38th.) Now the Parallelogram A B C D is double to the Triangle A B C, (*by the* 34th.) it is therefore double to the Triangle B C E.

USE.

Use 39. *The general method of measuring of the Area or Superficies of a Triangle, is grounded on this Proposition. Let the Triangle A B C be proposed, we draw from the Angle A the Line A D Perpendicular to the Base B C; and Multiplying the Perpendicular A D by the Semi-Base B E, the Product gives the Area of the Triangle; because Multiplying A D or E F by B E, we have the Content of the Rectangle B E F H, which is equal to the Triangle A B C. For the Triangle A B C is half of the Rectangle H B C G, (by the* 41st.) *as well as B E F H.*

We

We measure all sorts of Right Lined Figures, as ABCDE, Reducing them first into Triangles BCD, ABD, AED; by drawing the Lines AD, BD, and the Perpendiculars CG, BF, EI. For Multiplying the half of BD by CG, and the half of AD by BF, and by EI, we have the Area of all those Triangles; which added together is equal to the content of the Right Lined Figur ABCDE. Use 39.

We find the Area of Regular Polygons, by Multiplying one half of their peripheries by the Perpendicular drawn from their Centers to one of their Sides; for Multiplying IG by AG, we have the Rectangle HKLM equal to the Triangle AIB: And doing the same for every Triangle, taking always the Semi Base, we have the Rectangle HKON, whose Side KO Composed of the Semi-Bases, and consequently equal to the Semi-Periphery; and the Side HK equal to the Perpendicular IG. Use 39.

According to this Principle, Archimedes *hath Demonstrated that a Circle is equal to a Rectangle comprehended under the Semi-Diameter, and a Line equal to the Semi-Circumference.*

PROPOSITION XLII.
PROBLEM.

TO make a Parallelogram equal to a Triangle given, in an Angle equal to a Right Lined Angle given.

It is required to make a Parallelogram equal to the Triangle A B C, having an Angle equal to the Angle E. Divide the Base B C into two equal parts in D. Draw A G Parallel to B C, (*by the* 31ſt.) make alſo the Angle C D F equal to the Angle E, (*by the* 23^{d}.) Then draw the Parallel C G; the figure F D C G is a Parallelogram, ſeeing the Lines F G, D C; D F, C G, are Parallel. The Angle C D F is equal to the Angle E. I ſay that the Parallelogram is equal to the Triangle A B C.

Demonſtration. The Triangle A D C is half of the Parallelogram F D C G, (*by the* 41ſt.) it is alſo half the Triangle A B C, becauſe the Triangles A D C, A D B, are equal (*by the* 37th.) Therefore the Triangle A B C is equal to the Parallelogram F D C G.

PRO-

PROPOSITION XLIII.

PROBLEM.

THe complements of a Parallelogram are equal.

In the Parallelogram ABDC, the complements AFEH, EGDI, are equal.

Demonſtration. The Triangles ABC, BCD are equal (by the 34d:) Whence if we Subſtract the Triangles HBE, BIE; FEC, CGE, which are alſo equal (*by the ſame*;) the complements AFEH, GDIE, which remains, ſhall be equal.

PROPOSITION XLIV.

PROBLEM.

UPon a Right Line given, to make a Parallelogram at a Right Lined Angle given, equal to a Triangle given.

It is propoſed to make a Parallelogram having one of its Sides equal to the Line D, and one of its Angles equal to the Right Lined Angle E, which muſt be equal to the Triangle ABC.

A B C. Make by the preceding Problem the Parallelogram B F G H, having the Angle H B F equal to the Angle E, and which may be equal to the Triangle A B C. Continue the Sides G H, C F, and make H I equal to the Line D; then draw the Line I B N, and by the Points I and N, draw I L Parallel to G N, and N L Parallel to G I; and continue H B to M, and F B to K. The Parallelogram K L B M is the Parallelogram required.

Demonstration. The Angle H B F equal to the given Angle E, is also equal to the Angle K B M (*by the* 15.) and the Side K B is equal to the Line H I or D. Lastly, the Parallelogram M K is equal (*by the foregoing*) to the Parallelogram G F B H; and this was made equal to the Triangle A B C. Therefore the Parallelogram M K is equal to the Triangle A B C.

Q. E. F.

Use 44. *IN this Proposition is contained a sort of Geometrical Division. For in Arithmetical Division there is proposed a number which may be imagined to be like a Rectangle. For example, the Rectangle*

A B

AB containing Twelve square Feet, which is to be Divided by another Number, as by two; that is to say, that there must be made another Rectangle equal to the Rectangle AB, which may have BD too, for one of its Sides, and to find how many Feet the other Side ought to be, that is to say the Quotient. One may attain it Geometrically by a Rule and Compass. Take BD of the Length of Two Feet, and draw the Diagonal DEF; the Line AF is that which you seek for. For having made the Rectangle DCFG, the Complements EG, EC, are equal (by the 43d.) and EG hath for one of its Sides EH equal to BD two Foot, and EI equal to AF. This way of Division is called Application, because the Rectangle AB is applyed to the Line BD or EH; and this is the reason why Division is called Application; for the Ancient Geometricians did choose rather to make use of a Ruler and Compass, than Arithmetick.

PRO.

PROPOSITION XLV.
PROBLEM.

Upon a Right-Line to make a Parallelogram equal to a Right-Lined Figure given at a Right-Lined Angle given.

There is propoſed or given the Right Lined Figure ABCD, and it is required to make a Parallelogram equal thereto, having an Angle equal to the Angle E. Reduce the Right Lined Figure into Triangles, by drawing the Right Line BD; and make (*by the* 42^{d},) a Parallelogram FGHI, having the Angle FGH equal to the Angle E, and being in content equal to the Triangle ABD. Make alſo (*by the* 44th,) a Parallelogram IHKL, equal to the Triangle BCD, and having a Line equal to IH, and the Angle IHK equal to the Angle E. The Parallelogram FGKL ſhall be equal to the Right Lined Figure ABCD.

Demonſtration. It remains to prove that the Parallelograms FGHI, HKLI, are but one; that is to ſay that GH, HK make but one ſtreight Line

Line, the Angles G H I, and K, are equal to the Angle E; the Angle K and K H I are equal to Two Right, ſeeing we have made a Parallelogram K H I L. Whereof the Angles G H I, K H I, are equal to Two Right, and *(by the* 14th,*)* G H, H K, is one ſtreight Line.

USE.

This Propoſition is in the Practice much like to the former, and ſerveth to Meaſure the Content of any Figure whatſoever, by Reducing it firſt into Triangles, then making a Right Angled Parallelogram equal thereto. We may alſo make a Right Angled Parallelogram on a determined or given Line, and which ſhall be equal to ſeveral irregular Figures. Likewiſe having ſeveral Figures, we may make another equal to their difference.

PROPOSITION XLVI.
PROBLEM.

Upon a Right Line to describe a Square.

To describe a Square on the Right Line A B, erect two Perpendiculars A C, B D, equal to A B, and draw the Line B D.

Demonstration. The Angles at A and B being Right, the Lines A C, B D, are Parallel (*by the* 28th.) they are also equal. Therefore the Lines A B, C D, are Parallel and equal (*by the* 33d.) and the Angles at A and C, B and D, equal to Two Right (*by the* 29th.) and seeing A and B are Right Angles, the Angles C and D shall be also Right. Thence the Figure A D hath all its Sides equal, and all its Angles Right, and consequently is a Square.

PROPOSITION XLVII.
THEOREM.

The Square of the Base of a Right Angled Triangle is equal to the Sum of the Squares of the other two Sides.

It is supposed that the Angle B A C is

is a Right Angle; and that there be described Squares on the several Sides B C, A B, A C; the Square of the Base B C shall be equal to the Squares of the other two Sides A B, A C. Draw the Line A H Parallel to B D, C E, and draw also the Lines A D, A E, F C, B G, I prove that the Square A F is equal to the Right Angled Figure or long square B H, and the Square A G, to the Right Angled Figure C H; and that so the Square B E is equal to the Two Squares A F, A G.

Demonstration. The Triangles F B C, A B D, have their Sides A B, B F; B D, B C, equal; and the Angles F B C, A B D, are equal: Seeing that each of 'em besides the Right Angle includes the Angle A B C. Thence (*by the* 4th.) the Triangles A B D, F B C, are equal. Now the Square A F is double to the Triangle F B C, (*by the* 41st.) because they have the same Base B F, and are between the same Parallels B F, A C. Likewise the Right Lined Figure B H is double to the Triangle A B D, seeing they have the same Base B D, and are between the same Parallels B D, A H. Therefore the Square A F is equal to

the

the Right Lined Figure B H. After the same manner the Triangles A C E, G C B, are equal (*by the* 4th;) the Square A G is double the Triangle B C G, and the Right Lined Figure C H is double the Triangle A C E, (*by the* 41st.) Thence the Square A G is equal to the Right Lined Figure C H, and by consequence the Sum of the Squares A F, A G, are equal to the Square B D E C.

USE.

Use 47. *IT is said that* Pythagoras *having found this Proposition, Sacrificed One Hundred Oxen, in thanks to the Muses; it was not without reason, seeing this Proposition serves for a Foundation to a great part of the Mathematicks. For in the First place, Trigonometry cannot be without it, because it is necessary to make the Table, of all the Lines that can be drawn within a Circle, that is to say, of Chords, of Sines. Also Tangents and Secants; which I shall here shew by one Example.*

Let it be supposed that the Semi-Diameter A B be Divided into 10000 *parts, and that the Arch B C is* 30 *degrees.*

Seeing

Seeing the Chord or ſubtendent of 60 *Degrees, is equal to the Semi-diameter* A C; B D *the Sine of* 30 *degrees ſhall be equal to the half of* A C; *it ſhall therefore be* 5000, *in the Right Angled Triangle* A D B. *The Square of* A B *is equal to the Squares of* B D *and* A D; *make then the Square of* A B, *by Multiplying* 10000 *by* 10000; *and from that Product Subtract the Square of* B D 5000; *there remains the Square of* A D, *or* B F, *the Sine of the Complement, and extracting the Square Root, there is found the Line* F B. *Then, if by the Rule of Three, you ſay as* A D *is to* B D, *ſo is* A C *to* C E, *you ſhall have the Tangent* C E; *and adding together the Squares of* A C, C E, *you ſhall have* (by the 47th.) *the Square of* A E; *and by extracting the Root thereof, you ſhall have the Length of the Line* A E *the Secant.*

Uſe 47.

We augment Figures as much as we pleaſe by this Propoſition: Example, to double the Square A B C D, *continue the Side* C D, *and make* D E *equal to* A D; *the Square of* A E *ſhall be the double of the Square of* A B C D, *ſeeing that* (by the 47th.) *it is equal to the Squares of* A D *and* D E. *And making a Right*

Angle

Angle AEF, and taking EF equal to AB, the Square of AF shall be Triple to ABCD. And making again the Right Angle AFG, and FG, equal to AB, the Square of AG shall be Quadruple to to ABCD. What I have say of a Square, is to be understood of all Figures which are alike, that is to say, of the same species.

PROPOSITION XLVIII.
THEOREM.

IF the Two Squares made upon the Side of a Triangle be equal to the Square made on the other Side, then the Angle comprehended under the Two other Sides of the Triangle, is a Right Angle.

If the Square of the Side NP is equal to the Squares of the Sides NL, LP, taken together, the Angle NLP shall be a Right Angle, draw LR Perpendicular to NL, and equal to LP; then draw the Line NR.

Demonstration. In the Right Angled Triangle NLR, the Square of NR is equal to the Squares of NL, and of LR, or LP, (*by the* 47th.) now the Square of NP is equal to the same Squares of NL, LP; therefore the square of NR is equal to that of NP;

NP; and by consequence the Lines NR, NP, are equal. And because the Triangles NLR, NLP, have each of them the Side NL common; and that their Bases RN, NP, are also equal; the Angles NLP, NLR, shall be equal (*by the* 8th.) and the Angle NLR being a Right Angle, the Angle NLP, shall be also a Right Angle.

The End of the First Book.

THE

THE SECOND BOOK OF Euclid's *Elements*.

EUclid *Treateth in this Book of the Power of Streight Lines, that is to say of their Squares comparing the divers Rectangles which are made on a Line Divided, as well with the Square as with the Rectangle of the whole Line. This part is very useful, seeing it serveth for a Foundation to the Practical Principles of Algebra. The Three first Propositions Demonstrateth the Third Rule of Arithmetick; The Fourth teacheth us to find the Square Root of any number whatsoever; those which follow unto the Eighth, serveth in several accidents happening in Algebra: The remaining Propositions se*

the

the end of this Book, are conversant in Trigonometry. This Book appeareth at the first sight very difficult, because one doth imagine that it contains mysterious or intricate matters; notwithstanding the greater part of the Demonstrations are founded on a very evident Principle. viz. That the whole is equal to all its parts taken together, therefore one ought not to be discouraged, although one doth not Apprehend the Demonstrations of this Book at the First Reading.

DEFINITIONS.

A Rectangular Parallelogram is Comprehended under Two Right Lines, which at their Intersection containeth a Right Angle. Def. 1. of the Second Boook.

It is to be noted henceforward, that we call that Figure a Rectangular Parallelogram, which hath all its Angles Right; and that the same shall be distinguished as much as is requisite, if we give thereto Length and Breadth, naming only Two of its Lines which comprehendeth any one Angle; as the Lines A B, B C, For the Rectangular Parallelogram A B C D is comprehended under the Lines

Lines A B, B C; having B C for its Length, and A B for its Breadth, whence it is no necessary to mention the other Lines, because they are equal to those already spoken of. I have already taken notice that the Line A B being in a Perpendicular Position in respect of B C, produceth the Rectangle A B C D, if moved along the Line B C; and that this Motion Representeth Arithmetical Multiplication, in this manner, as the Line A B moves along the Line B C, that is to say taken as many times as there are Points in B C, Composeth the Rectangle A B C D; wherefore Multiplying A B by B C, I shall have the Rectangle A B C D. As suppose I know the Number of Mathematical Points there be in the Line A B, for Example, let there be 40, and that in B C there be 60 Points, it is evident that the Rectangle A B C D shall have as many Lines equal to A B as there are Points in B C, and that by Multiplying 40 by 60, the Product will be 2400, the Number of Mathematical Points contained in the Rectangle A B C D.

I

I may take what quantity I please for a Mathematical Point, provided I do not subdivide the same; but it is to be noted, that when I measure a Line, I take for a Mathematical Point the Measure which I make use of in measuring the same: For Example, when I say a Line of Five Foot, I take a Line of one Foot to be a Mathematical Point, without considering that the same is composed of parts. In like-manner if I measure a Superficies, the measure I make use of is to me a known Superficie; for example, a Square Foot, which I do not subdivide. A Square is rather made use of than any other Figure; because a Square hath its Length equal to its Breadth; whence it is, one is not confined to mention both its dimensions, when I would determine the Area of the Rectangle A B C D, I conceive not its sides as if they were single Lines, but as Rectangles of a determined Breadth: For Example, when I say that the Rectangle A B C D hath the Side A B of Four Foot in Length; seeing that a Foot is as a Mathematical Point, I conceave the Side A B as being one Foot in Breadth; and as the Rectangle A B E F. So that by knowing how many times the Breadth B E is contained in B C, I shall know how many

times the Line AB is in the Rectangle ABCD; that is to say, Multiplying AB, which is Four Foot by BC 6, I shall have 24 Square Foot. Likewise by knowing the superficial content of the Rectangle ABCD, which is 24 Square Foot, and one of its Sides AB 4, Dividing the same by 4, giveth the other Side BC 6 Foot.

Having drawn the Diameter of a Rectangle, one of the lesser Rectangles through which the Diameter passeth, together with the Two Complements, is called a Gnomon, as if one should draw the Diameter BD, the Rectangle EG, through which passeth the Diameter, together with the Complements EF, GH, is called A Gnoman because of its Figure, which is that of a Carpenters Square.

PRO-

PROPOSITION I.
PROBLEM.

IF Two Right Lines be given, and one of them be Divided into as many parts or Segments as you pleaſe, the Rectangle comprehended under the Two whole Right Lines, ſhall be equal to all the Rectangles contained under the Line which is not Divided, and the ſeveral Segments of the divided Line.

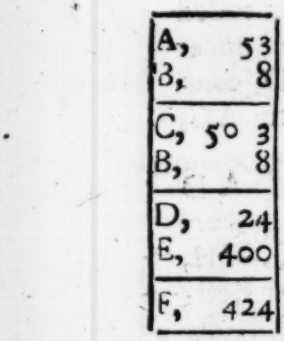
Let the Lines propoſed be A B, A C; and let A B be Divided into as many parts as you pleaſe, the Rectangle A D comprehended under the Lines A B, A C, is equal to the Rectangle A G, comprehended under A C, A E; and to the Rectangle E H comprehended under E G, equal to A C, and under E F; and to the Rectangle F D, comprehended under F H equal to A C, and under F B.

Demonſtration. The Rectangle A D is equal to all its parts taken together, which are the Rectangles A G, E H, and F D. There being no other Rectangle remaining, therefore the Rectangle A D is equal to the Rectangles A G, E H, F D, taken together.

Arithmetically or by Numbers.

THe ſame Propoſition is verified by Numbers. Let us ſuppoſe that the Line AC is in Length Five Foot, AE two Foot, EF Four, FB three; and by conſequence AB Nine, the Rectangle comprehended under AC Five, and AB Nine, that is to ſay Five times Nine or Forty Five, is equal to twice Five or Ten; and Four times Five or Twenty, and Three times Five or Fifteen for Ten, Twenty, and Fifteen, added together is equal to Forty Five.

USE.



THis Propoſition Demonſtrateth the ordinary practice of Multiplication. For example, let the number A, 53, *be to be Multiplyed by the Line AB, repreſented by the Number* B, 8. *I cut or ſeperate the number* A, *into as many parts as it hath characters: For Example, into* 50 *and* 3; *which I Multiply by* 8, *ſaying* 8 *times* 3 *is* 24; *and by*

by ſo doing I make a Rectangle. Again I Multiply the Number 50 *by* 8; *the Product ſhall be E*, 400. *It is evident, that the Product of* 8 *times* 53, *which is* 424, *is equal to the Product of* 24, *and to the Product of* 400, *they being added or taken together.*

PROPOSITION II.

THEOREM.

THe Square of a Line is equal to all the Rectangles comprehended under the whole, and under its parts.

Let A B be the propoſed Line, and its Square A B D C. I ſay that the Square A B D C is equal to a Rectangle comprehended under the whole Line A B, and under A E; and to a Rectangle comprehended under A B, E F; and to a Third comprehended under A B, and F B.

Demonſtration. The Square A B D C is equal to all its parts taken together, which are the Rectangles AG, EH, FD. The firſt of which is comprehended under A C equal to A B, and under A E. The Second E H, is comprehended under E G equal to A C or A B,

and under E F. And the Third F D is comprehended under F H, equal to A B, and under F B: And it being the same thing to be comprehended under a Line equal to A B, as to be comprehended under A B it self. Therefore the Square of A B is equal to all the Rectangles comprehended under A B, and under A E, E F, F B, parts of A B.

ARITHMETICALLY.

LEt the Line A B represent the Number Nine; its Square shall be 81. Let the part thereof A E be Four; E F, Three; F B, Two: Ninetimes Four is Thirty Six, Nine times Three is Twenty Seven, and Nine times Two is Eighteen. It is evident that 36, 27, and 18, added together are 81.

USE.

THis Proposition serveth to prove Multiplication; as also for Algebriacal Equations.

PRO-

PROPOSITION III.
THEOREM.

IF a Line be Divided into Two parts, the Rectangle comprehended under the whole Line, and under one of its parts, is equal to the Square of the same part, and to the Rectangle comprehended under both its parts.

Let the Line A B be Divided into Two parts in the point C; and let there be made a Rectangle comprehended under A B, and one of its parts; for example A C, that is to say, that A D be equal to A C; and compleat of the Rectangle A F, which shall be equal to the Square of A C, and to the Rectangle comprehended under A C, C B. Draw the Perpendicular C E.

Demonstration. The Rectaugle A F, comprehended under A B, and under A D equal to A C, is equal to all its parts, which are the Rectangles A E, C F. The first A E is the Square of A C, seeing that the Lines A C, A D, are equal; and the Rectangle C F is comprehended under C B, and under C E, equal to A D, or A C. Therefore the Rectangle comprehended under

A B, A C, Is equal to the Square of A C, and to a Rectangle comprehended under A C, C B.

ARITHMETICALLY.

LEt A B be 8, A C, 3; C B, 5; the Rectangle comprehended under A B and A C, shall be Three times 8, or 24; the square of A C, 3, is 9; the Rectangle comprehended under A C, 3; and C B 5, is 3 times 5, or 15. It is evident, that 15 and 9 are 24.

USE.

A	43	
C	40.	3
B,		3
120.		9.
129		

THis Proposition serveth likewise to Demonstrate the ordinary practice of Multiplication. For Example, if one would Multiply the Number 43 *by* 3, *having separated the Number of* 43 *into two parts in* 40, *and* 3; *three times* 43 *shall be as much as three times* 3, *which is Nine, the Square of Three; and Three times Forty, which is* 120; *for* 129 *is Three Times* 43. *Those which are young beginners ought not to be discouraged, if*

if they do not conceive immediately these Propoſitions, for they are not difficult, but becauſe they do imagine they contain ſome great Myſtery.

PROPOSITION IV.

THEOREM.

IF a Line be Divided into Two Parts, the Square of the whole Line ſhall be equal to the Two Squares made of its parts, and to Two Rectangles comprehended under the ſame parts.

Let the Line AB be Divided in C, and let the Square thereof A B D E be made; let the Diagonal E B be drawn, and the Perpendicular C F cutting the ſame; and through that Point let there be drawn G L Parallel to A B. It is evident, that the Square A B D E is equal to the Four Rectangles G F, C L, C G, L F. The Two firſt are the Squares of A C, and of C B; the Two Complements are comprehended under A C, C B.

Demonſtration. The Sides A E, A B, are equal; thence the Angles A E B, A B E, are half Right; and becauſe of the Parallels G L, A B, the Angles of

the Triangles of the Square G E, (*by the* 29th,) ſhall be equal; as alſo the Sides (*by the* 6th *of the* 1.) Thence GF is the Square of A C. In like manner the Rectangle C L is the Square of C B; the Rectangle G C is comprehended under A C, and A G equal to B L, or B C; the Rectangle L F is comprehended under L D equal to A C, and under F D equal to B C.

Coroll. If a Diagonal be drawn in a Square, the Rectangles through which it paſſeth are Squares.

USE.

A,	144
B,	22
C,	12

This Propoſition giveth us the practical way of finding or extracting the Square Root of a Number propounded. Let the ſame be the number A, 144, *repreſented by the Square A D, and its Root by the Line A B. Moreover I know that the Line required A B muſt have Two Figures. I therefore imagine that the Line A B is Divided in C, and that A C repreſenteth the firſt Figure; and B C the Second. I ſeek the Root of the Firſt Figure of the Number* 144, *which is* 100, *and I find that it is* 10; *and making its Square*

100,

100, *represented by the Square G F; I Subtract the same from* 144, *and there remains* 44 *for the Rectangles G C, F L, and the Square C L. But because this gnomonicall Figure is not proper, I transport the Rectangle F L, in K G, and so I have the Rectangle K L containing* 44. *I know also almost all the Length of the Side K B; for AC is* 10, *therefore K C is* 20: *I must then Divide* 44, *by* 20; *that is to say, to find the Divisor, I double the Root found, and I say how many times* 20 *in* 44? *I find it* 2 *times, for the Side B L; but because* 20 *was not the whole Side K B but only K C; this* 2 *which cometh in the Quotient is to be added to the Divisor, which then will be* 22. *So I find the same* 2 *times precisely in* 44, *the Square Root then shall be* 12. *You see that the Square of* 144 *is equal to the Square of* 10, *to the Square of* 2, *which is* 4, *and to twice* 20, *which are Two Rectangles comprehended under* 2, *and under* 10.

PRO-

PROPOSITION V.

THEOREM.

IF a Right Line be cut into equal parts, and into unequal parts; the Rectangle comprehended under the unequal parts, together with the Square which is of the middle part, or difference of the parts, is equal to the Square of half the Line.

If the Line A B is Divided equally in C, and unequally in D; the Rectangle A H comprehended under the Segments A D, D B, together with the Square of C D, shall be equal to the Square C F; that is of half of A B, *viz.* C B. Make an end of the Figure as you see it; the Rectangles L G, D I, shall be Squares (*by the Coroll. of the* 4th.) I prove that the Rectangle A H, comprehended under A D, and D H, equal to D B, with the Square L G, is equal to the Square C F.

Demonstration. The Rectangle A L, is equal to the Rectangle D F; the one and the other being comprehended under half the Line A B, and under B D, or D H equal thereto. Add to both the Rectangle C H; the Rectangle A H,

A H, ſhall be equal to the Gnomon L B G. Again to both add the Square L G; the Rectangle A H, with the Square L G, ſhall be equal to the Square C F.

ARITHMETICALLY.

LEt A B be 10; A C is 5, as alſo C B. Let C D be 2, and D B, 3; the Rectangle comprehended under A D, 7; and D B, 3: that is to ſay 21, with the Square of C D 2, which is 4, ſhall be equal to the Square of C B 5, which is 25.

USE.

THis Propoſition is very uſeful in the Third Book; we make uſe thereof in Algebra, to Demonſtrate the way of finding the Root of an affected Square or Equation.

PRO-

PROPOSITION VI.

THEOREM.

IF one add a Line to another which is Divided into Two equal parts; the Rectangle comprehended under the Line compounded of both, and under the Line added, together with the Square of half the Divided Line, is equal to the Square of a Line compounded of half the Divided Line, and the Line added.

If one add the Line B D, to the Line A B, which is equally Divided in C; the Rectangle A N, comprehended under A D, and under D N, or D B, with the Square of C B, is equal to the Square of C D. Make the Square of C D, and having drawn the Diagonal F D, draw B G Parallel to F C, which cuts F D in the Point H, through which passeth H N Parallel to A B: K G shall be the Square of B C; and B N, that of B D.

Demonstration. The Rectangles A K, C H, on equal Bases, A C, B C, are equal (*by the* 38.th *of the* 1st.) The Complements C H, H E, are equal (*by the* 43d *of the* 1st.) Therefore the Rectangles A K,

AK, HE, are equal. Add to both the Rectangle CN, and the Square KG, the Rectangles AK, CN, that is to say the Rectangle AN with the Square KG, shall be equal to the Rectangles CN, HE, and to the Square KG, that is to say to the Square CE.

Arithmetically or by Numbers.

LEt AB be 8, AC 4, CB 4, BD 3; then AD shall be 11. It is evident that the Rectangle AN, three times 11, that is to say 33 with the Square of KG 16, which together are 49, is equal to the Square of CD 7, which is 49, for 7 times 7 is 49.

USE.

MAurolycus *measured the whole Earth by one single Observation, making use of this Proposition. He would that one should observe from A the top of a Mountain whose height is known; the Angle BAC, which is made by the Line AB which toucheth the superficies of the Earth, and the Line AC which passeth through the Center; and that in the Triangle ADF; knowing the Angle A,* Fig. 6.

and

and the Right Angle *A D F*, one findeth by Trigonometry the Sides *A F*, *F D*; and because that it is easie to Demonstrate that *F D*, *F B*, are equal, one shall know the Length of the Line *A B*, and its Square. Now having Demonstrated by the preceding Proposition, that the Line *E D* being Divided into two equal parts in the Point *C*, and having added thereto *A D*; the Rectangle comprehended under *E A*, and under *A D*, together with the Square of *C D*, or *C B*, is equal to the Square of *A C*, and the Angle *A B C* being Right, (as it is proved in the Third Book) the Square of *A C* is equal to the Squares of *A B*, *B C*. Therefore the Rectangles *A E*, *A D*, together with the Square of *B C*, is equal to the Squares of *A B*, *B C*. Take from both the Square of *B C*; the Rectangle under *A E*, *A D*, shall be equal to the Square of *A B*. Divide then the Square of *A B*, which you know by the Height of the Mountain, which is *A D*; the Quotient shall be the Line *A E*, from which must be Subtracted the Height of the same Mountain; and you shall have *D E*, the Diameter of the Earth.

We make use of the same Proposition in Algebra; as also to Demonstrate the practice

practice of finding the Root of a Square equal to a Number encreaſed more by a certain number of Roots. The Two Propoſitions following ſerveth alſo to prove other like Practiſes.

PROPOSITION VII.
THEOREM.

IF one Divide a Line, the Square of the whole Line, together with that of one of its parts, ſhall be equal to Two Rectangles comprehended under the whole Line, and that firſt part, and to the Square of the other part.

Let the Line AB be Divided at diſcretion, in the Point C; the Square AD of the Line AB, together with the Square AL, ſhall be equal to Two Rectangles comprehended under AB, AC. With the Square of CB make the Square AB; and having drawn the Diagonal EB, and the Lines CF, HGI, prolong EA, and make AK equal to AC; ſo ſhall AL be the ~~Square of AC, and HK ſhall be equal~~ to AB. For HA is equal to GC; and GC is equal to CB, ſeeing CI is the Square of CB, (*by the Coroll. of the* 4th.)

Demonſtration.

Demonſtration. It is evident that the Squares AD, AL, are equal to the Rectangles HL, HD, and to the Square of CI. For the Rectangle HL is comprehended under HK equal to AB, and under KL equal to AC. In like manner, the Rectangle HD is comprehended under HI, equal to AB, and under HE equal to AC. Therefore the Squares of AB, AC, are equal to Two Rectangles comprehended under AB, AC, and to the Square of CB.

ARITHMETICALLY.

LEt it be ſuppoſed that the ſame AB contains 9 equal parts; AC, 4; CB, 5: the Square of AB, 9, is 81; the Square of AC 4, is 16. Which being added together is 97. A Rectangle under AB, AC, or 4 times 9 is 36; which doubled is 72; The Square of CB, 5, is 25. Now 25 and 72 is alſo 97.

PRO-

PROPOSITION VIII.
THEOREM.

IF one Divide a Line, and that thereto one add one of its parts, the Square of the Line Compoſed ſhall be equal to Four Rectangles, comprehended under the Firſt Line, and under the part added together with the Square of the other part.

Let the Line AB be Divided at diſcretion in the Point C; and let thereto be added BD, equal to CB, the Square of AD ſhall be equal to 4 Rectangles comprehended under AB, BC or BD, and to the Square of AC. Let the Square of AD be made, and having drawn the Diagonal AE, draw the Perpendiculars BP, CN, which cut the Diagonal in L and O; draw alſo the Lines MIH, GLR, Parallel to AB; the Rectangles GC, IK, PH, MB, NR, ſhall be Squares (*by the Coroll. of the* 4th.)

Demonſtration. The Square ADEF is equal to all its parts; the Rectangles IB, OD, PM, are comprehended under Lines equal to AB and CB. If you add the Rectangle ML to the Rectangle PH; you ſhall have a Rectangle compre-

comprehennded under one Line equal to A B, and under another equal to C B or B D. There remains only the Square G C, which is that of A C. Therefore the Square of A D is equal to Four Rectangles comprehended under A B, B D, and to the Square of A C.

ARITHMETICALLY.

LEt the Line A B contain 7 parts; A C 3, C B 4, as well as B D; the Square of A D, 11, shall be 121. A Rectangle under A B, 7, and B D, 4; is Twenty Eight: which taken 4 times, is 112. and the Square of 3 is 9. Now 112. and 9. added are 121.

PROPOSITION IX.

THEOREM.

IF a Line be equally Divided, and unequally; the Squares of the unequal parts shall be double to the Squares of half the Line, and of the part between.

Let the Line A B be Divided equally, in the Point C; and unequally, in the Point D. The Squares of the unequal parts A D, D B, shall be double of the Squares

Squares of A C, which is the half of A B, and the Square between both C D. Draw A B, and the Perpendicular C E, equal to A C: Draw alſo the Lines A E, B E, and the Perpendicular D F; as alſo F G Parallel to C D. Then draw the Line A F.

Demonſtration. The Lines A C, C E, are equal, and the Angle C is Right; thence (*by the* 6th *of the* 1ſt.) the Angles C A E, C E A, are equal to a half Right Angle. In like manner, the Angles C E B, C B E, C F E, D F B, are half Right; the Lines G F, G E, D F, D B, are equal; and the total Angle A E F is Right. The Square of A E (*by the* 47th *of* 1ſt.) is equal to the Squares of A C, C E, which are equal: Thence it is double to the Square of A C. After the ſame manner the Square of E F is double to the Square of G F, or C D: Now the Square of A F is equal to the Squares of A E, E F; ſeeing that the Angle A E F is Right: Therefore the Square of A F is double to the Squares of A C, C D. The ſame Square A F is equal to the Squares of A D, D F, or D B; ſeeing that the Angle D is Right: Therefore the Squares of A D, D B, are double

double to the Squares of AC, CD.

ARITHMETICALLY.

LEt AB be 10; AC, 5; CD, 3; DB, 2: The Squares of AD, 8, and DB, 2; that is to say 64, and 4, which together are 68, are double to the Square AC, 5, which is 25; and of the Square CD, 3, which is 9; for 25 and 9 are 34, which doubled make 68.

USE.

I *Have not found this Proposition, nor the insuing, but only in Algebra.*

PROPOSITION X.

THEOREM.

IF *one add a Line, to another which is* equally *Divided; the Square of the Line Composed of both, with the Square of the Line added, are double to the Square of half the Line, and to the Square which is Composed of the half Line and the Line added.*

If

If one ſuppoſeth A B, to be Divided in the middle at the Point C; and if thereto be added the Line B D, the Squares of A B and B D, ſhall be double to the Squares of A C, and C D, added together. Draw the Perpendiculars C E, D F, equal to A C: Then draw the Lines A E, E F, A G, E B G.

Demonſtration. The Lines A C, C E, C B, being equal, and the Angles at the Point C being Right: The Angles A E C, C E B, C B E, D B G, D G B, ſhall be half Right; and the Lines D B, D G, and E F, F G, C D, ſhall be equal. The Square of A E, is double to the Square of A C; the Square of E G, is double to the Square of E F, or C D, (*by the* 47th *of the* 1ſt.) Now the Square of A G, is equal to the Squares of A E, E G, (*by the* 47th *of the* 1ſt:) Therefore the Square of A G is double to the Squares of A C, C D. The ſame A G (*by the* 47th *of the* 1ſt,) is equal to the Squares of A D, B D, or G D: Therefore the Squares of A D, B D, are double the Squares of A C, C D.

ARITH-

ARITHMETICALLY.

LEt A B be 6 parts, A C, 3; C B, 3; B D, 4: the Square of A D, 10, is 100. the Square of B D, 4, is 16, which are 116. The Square of A C, 3, is 9; the Square of C D, 7, is 49. Now 49. and 9, is 58, the half of 116.

PROPOSITION XI.

PROBLEM.

TO Divide a Line, ſo that the Rectangle comprehended under the whole Line, and under one of its parts, ſhall be equal to the Square of the other part.

It is propoſed to Divide the Line AB, ſo that the Rectangle comprehended under the whole Line A B, and under H B, be equal to the Square of A H. Make a Square of A B (*by the* 46.*th of the* 1*ſt*:) Divide A D in the middle in E; then Draw E B, and make E F equal to E B. Make the Square A F, that is to ſay that A F, A H, be equal. I ſay that the Square of A H, ſhall be equal to the Rectangle H C, comprehended

hended under H B, and the Line B C, equal to A B.

Demonſtration. The Line A D is equally divided in E, and there is added thereto the Line F A; thence (*by the* 6th.) the Rectangle D G, comprehended under D F and F G, equal to A F, with the Square of A E, is equal to the Square of E F, equal to E B. Now the Square of E B is equal to the Squares of A B, A E, (*by the* 47th *of the* 1ſt.) therefore the Squares of A B, A E, are equal to the Rectangle D G, and to the Square of A E: and taking away from both, the Square of A E, the Square of A B, which is A C, ſhall be equal to the Rectangle D G: taking alſo away the Rectangle D H, which is in both, the Rectangle H C ſhall be equal to the Square of A G.

USE.

THis Propoſition ſerveth to cut a Line in extream and mean Proportion; as ſhall be ſhewn in the Sixth Book. It is uſed often in the 14th. *of* Euclid's *Elements, to find the Sides of Regular Bodies. It ſerveth for the* 10th. *of the Fourth Book, to inſcribe a Pentagone in a Circle, as al-*

ſo a Pentadecagone. You ſhall ſee other uſes of a Line thus divided, in (the 30th. of the Sixth Book)

PROPOSITION XII.

THEOREM.

IN an obtuſe angled Triangle the Square of the ſide oppoſite to the obtuſe Angle, is equal to the Squares of the other two ſides, and to two Rectangles comprehended under the ſide on which one draweth a Perpendicular, and under the Line which is between the Triangle, and that perpendicular

Let the Angle A C B of the Triangle A B C, be obtuſe; and let A D be drawn perpendicular to B C. The Square of the ſide A B, is equal to the Squares of the ſides A C, C B, and to two Rectangles comprehended under the ſide B C, and under D C.

Demonſtration. The Square of A B is equal to the Squares of A D, D B. (*by the* 47th. *of the* 1ſt.) the Square of D B is equal to the Squares of D C and C B, and to two Rectangles comprehended under D C, C B, (*by the* 4th.) therefore the Square of A B is equal to

the

the Squares of A D, D C, C B, and to two Rectangles comprehended under D C, C B, (in the place of the two laſt Squares A D, D C. Put the Square of A C, which is equal to them (*by the* 47th *of the* 1ſt.) the Square of A B ſhall be equal to the Square of A C and C B. and to two Rectangles comprehended under D C, C B.

USE.

THis Propoſition is uſeful to meaſure the Area of a Triangle, its three ſides being known: for Example; If the ſide A B was twenty Foot; A C, 13; B C, 11; the Square of A B would be four hundred; the Square of A C, one hundred ſixty nine; and the Square of B C, one hundred twenty one: the Sum of the two laſt is, Two hundred and ninety; which being ſubtracted from four hundred, leaves one hundred and ten for the two Rectangles under B C, C D, the one half fifty five ſhall be one of thoſe Rectangles: which divided by B C 11, we ſhall have five for the Line C D, whoſe Square is twenty five; which being ſubtracted from the Square of A C, one hundred ſixty nine, there remains the Square of A D, one

hundred forty four; and its Root ſhall be the ſide A D: which being multiplied by $5\frac{1}{2}$, *the half of B C, you have the Area of the Triangle A B C, containing 66 ſquare Feet.*

PROPOSITION XIII.

THEOREM.

IN any Triangle whatever, the Square of the ſide oppoſite to an acute Angle, together with two Rectangles comprehended under the ſide on which the Perpendicular falleth, and under the Line which is betwixt the Perpendicular and that Angle, is equal to the Square of the other ſides.

Let the propoſed Triangle be A B C, which hath the Angle C, acute; and if one draw A D perpendicular to B C, the Square of the ſide A B, which is oppoſite to the acute Angle C, together with two Rectangles comprehended under B C, D C, ſhall be equal to the Squares of A C, B C.

Demonſtration. The Line B C is divided in D; whence (*by the* 7th.) the Square of B C, D C, are equal to two Rectangles under B C, D C, and to the

the Square of B D: add to both the Square of A D; the Square of B D, D C, A D, ſhall be equal to two Rectangles under B C, D C; and to the Squares of B D, A D, in the place of the Squares of C D, A D, put the Square of A C, which is equal to them (*by the* 47th. *of the* 1ſt.) and inſtead of the Squares of B D, A B, ſubſtitute the Square of A B, which is equal to them, the Squares of B C, A C, ſhall be equal to the Square of A B, and to two Rectangles comprehended under B C, D C.

USE.

THeſe Propoſitions are very neceſſary in Trigonometry: I make uſe thereof in the eighth Propoſition of the third Book, to prove, That in a Triangle there is the ſame Reaſon between the whole Sine and the Sine of an Angle, as are between the Rectangle of the ſides comprehending that Angle, and the double Area of the Triangle. I make uſe thereof in ſeveral other Propoſitions, as in the ſeventh.

PROPOSITION XIV.

PROBLEM.

TO describe a Square equal to a right lined Figure given.

To make a Square equal to a Right lined Figure A, make (*by the* 45th. *of the* 1ſt.) a Rectangle B C, D E, equal to the Right lined Figure A: if its ſides C D, D C, were equal, we ſhould have already our deſire; if they be unequal, continue the Line B C, until C F be equal to C D; and dividing the Line B F in the middle, in the Point G deſcribe the Semicircle F H B, then continue D C to H, the Square of the Line C H is equal to the Right lined Figure A: draw the Line G H.

Demonſtration. The Line B F is equally divided in G, and unequally in C; thence (*by the* 5th.) the Rectangle comprehended under B C, C F, or C D, that is to ſay, the Rectangle B D, with the Square C G, is equal to the Square of G B, or to its Equal G H. Now (*by the* 47th. *of the* 1ſt.) the Square of G H is equal to the Square of C H, C G, therefore the Rectangle B D, and the Square

Square of C G, is equal to the Squares of C G, and of C H; and taking away the Square of C G, which is common to both, the Rectangle B D, or the Right lined Figure A, is equal to the Square of C H.

USE.

THis Propoſition ſerveth in the firſt place, to reduce into a Square any Right lined Figure whatever: and whereas a Square is the firſt Meaſure of all Superficies, becauſe its Length and Breadth is equal, we meaſure by this means all right lined Figures. In the ſecond place this Propoſition teacheth us to find a mean Proportion between two given Lines; as we ſhall ſee in the Thirteenth Propoſition of the Sixth Book.

This Propoſition may alſo ſerve to ſquare curve lined Figures, and even Circles themſelves; for any crooked or curve lined Figure may, to ſence, be reduced to a Right lined Figure; as if we inſcribe in a Circle a Polygon having a thouſand ſides, it ſhall not be ſenſibly different from a Circle: and reducing the Polygone into a Square, we ſquare nearly the Circle.

THE THIRD BOOK OF Euclid's *Elements*.

THis Third Book explaineth the Propriety of a Circle, and compareth the divers Lines which may be drawn within and without its Circumference. It farther conſidereth the Circumſtances of Circles, which cut each other, or which touch a ſtreight Line; and the different Angles which are made, as well thoſe in their Centers as in their Circumferences. In fine, it giveth the firſt Principles for eſtabliſhing the Practice of Geometry; by the which we make uſe, and that very commodiouſly, of a Circle in almoſt all Treatiſes in the Mathematicks.

DEFI-

DEFINITIONS.

1. Thoſe Circles are equal, whoſe Diameters, or Semidiameters, are equal. Def. of the 3 Book.

2. A Line toucheth a Circle when meeting with the Circumference thereof it cutteth not the ſame: as the Line A B. Fig. 1.

3. Circles touch each other, when Meeting they cut not each other; *as the Circles* A B *and* C. Fig. 2.

4. Right Lines in a Circle are equally diſtant from the Center, when Perpendiculars drawn from the Center to thoſe Lines are equal. *As if the Lines E F, E G, being Perpendiculars to the Lines A B, C D, are equal, A B, C D, ſhall be equally diſtant from the Center; becauſe the Diſtance ought always to be taken (or meaſured) by Perpendicular Lines.* Fig. 3.

5. A Segment of a Circle is a Figure terminated on the one ſide by a ſtreight Line, and on the other by the Circumference of a Circle; *as* L O N, L M N. Fig. 4.

6. The Angle of a Segment is an Angle which the Circumference maketh

keth with a ſtreight Line; *as the Angle* O L N, I M N.

Fig. 5. 7. An Angle is ſaid to be in a Segment of a Circle, when the Lines which form the ſame are therein; *as the Angle* F G H, *is in the Segment* F G H.

Fig 6. 8. An Angle is upon that Arch to which it is oppoſite, or to which it ſerveth for a Baſe; *as the Angle* F G H, *is upon the Arch* F I H, which may be ſaid to be its Baſe

Fig. 6. 9. A Sector is a Figure comprehended under two Semidiameters, and under the Arch which ſerveth to them for a Baſe; *as the Figure* F I G H.

PROPOSITION I.

PROBLEM.

To find the Center of a Circle.

IF you would find the Center of the Circle A E B D, draw the Line A B, and divide the ſame in the middle, in the Point C; at which Point erect a Perpendicular E D, which you ſhall divide alſo equally in the Point F. This Point F ſhall be the Center of the Circle; for if it be not, imagine, if you pleaſe, that

that the Point G is the Center: draw the Lines G A, G B, G C.

Demonſtration. If the Point G were the Center, the Triangles G A C, G B C, would have the ſides G A, G B, *equal by the definition of a Circle*; A C, C B, are equal to the Line A B, having been divided in the middle, in the Point C. And C G being common, the Angles G C B, G C A, would then be equal (*by the* 8th. of the 1ſt.) and C G would be then a Perpendicular. and not C D; which would be contrary to the Hypotheſis. Therefore the Center cannot be out of the Line C D. I further add, that it muſt be in the Point F, which divideth the ſame into two equal Parts; otherwiſe the Lines drawn from the Center to the Circumference would not be equal.

Corollary. The Center of a Circle is in a Line which divideth another Line in the middle, and that perpendicularly.

USE.

THis firſt Propoſition is neceſſary to demonſtrate thoſe which follow.

PRO-

PROPOSITION II.

THEOREM.

A Streight Line drawn from one point of the Circumference of a Circle to another, ſhall fall within the ſame.

Let there be drawn from the Point B, in the Circumference, a Line to the Point C. I ſay that it ſhall fall wholly within the Circle. To prove that it cannot fall without the Circle, as B V C; having found the Center thereof, which is A, draw the Lines A B, A C, A V.

Demonſtration. The Sides A B, A C, of the Triangle A B C, are equal: whence (*by the* 5th. *of the* 1ſt.) the Angles A B C, A C B, are equal. And ſeeing the Angle A V C, is exteriour in reſpect of the Triangle A V B, it is greater than A B C, (*by the* 16th. *of the* 1ſt.) it ſhall be alſo greater than the Angle A C B. Thence (*by the* 19th *of the* 1ſt.) in the Triangle A C V, the ſide A C, oppoſite to the greateſt Angle A V C, is greater than A V; and by conſequence, A V cannot reach the circumference of the Circle, ſeeing it

is.

is ſhorter than A C, which doth but reach the ſame; wherefore the Point V is within the Circle: the ſame may be proved of any Point in the Line A B; and therefore the whole Line A B falls within the Circle.

USE.

IT is on this Propoſition that are grounded thoſe which demonſtrate that a Circle toucheth a ſtreight Line but only in one Point: for if the Line ſhould touch two Points of its Circumference, it would be then drawn from one Point of its Circumference to the other; and conſequently, would fall within the Circle, according to this Propoſition; although its Definition ſaith, that it cutteth not its Circumference. Theodoſius *demonſtrateth after the ſame manner, That a Globe cannot touch a Plain but in one ſingle Point; otherwiſe the Plain would enter into the Globe.*

PRO-

PROPOSITION III.

THEOREM.

IF the Diameter of a Circle cutteth a Line which paſſeth not through the Center, into two equal Parts, it ſhall cut the ſame at right Angles; and if it cutteth it at Right Angles it divideth it into two equal Parts.

If the Diameter A C cut the Line B D, which paſſeth not through the Center F, into two equal Parts in E, it ſhall cut the ſame at Right Angles. Draw the Lines F B, F D.

Demonſtration. In the Triangles F E B, F E D, the ſide F E is common; the ſides B E, E D, are equal, ſeeing the Line B D is divided equally in the Point E; the Baſes F B, F D, are equal: thence (*by the* 8th. *of the* 1ſt.) the Angles B E F, D E F, are equal, and conſequently right. Moreover, I ſay, that if the Angles B E F, D E F, are right, the Line B D ſhall be divided into two equal Parts in E; that is to ſay, the Lines B E, E D, ſhall be equal.

De-

Demonſtration. The Triangles B E F, D E F, are Rectangular: thence (*by the 47th. of the* 1st.) the Square of the ſide F D ſhall be equal to the Squares of E D, and of E F: in like manner, the Square of B F ſhall be equal to the Squares of B E, E F. Now the Squares of B F, F D, are equal, becauſe the Lines are equal: therefore the Squares of B F, E F, are equal to the Squares of E D, E F; and taking away E F, the Squares of B E, E D, ſhall be equal, and conſequently the Lines.

PROPOSITION IV.

THEOREM.

TWo Lines drawn in a Circle, cut not each other equally, if the Point of their Interſection be not in the Center.

If the Lines A C, B D, cut each other in the Point I, which is not the Center of the Circle, they will not cut each other equally. In the firſt place, if one of them ſhould paſs through the Center, it is evident, that the ſame cannot be divided into two equal Parts, but by the Center. If neither of them paſs through the Center, as B D, E I, draw the Line A I C, which paſſeth through the Center. De-

Demonſtration. If the Line A C divide the Line B D into two equal Parts in I, the Angles A I D, A I B, would be equal (*by the* 2d.) In like manner, if the Line E G was divided into two equal Parts, in the Point I, the Angle A I G would be right: thence the Angles A I B, A I G, would be right, and conſequently equal, which cannot be, the one being part of the other. Laſtly, the Line A I C, which paſſeth through the Centers, would then be perpendicular to the Lines B D, G I; if both of them were divided equally in the Point I.

U S E.

Both of thoſe Propoſitions are of uſe in Trigonometry: hereby is demonſtrated, that the half Chord of an Arch is perpendicular to the Semidiameter; and by conſequence, it is the ſine of the half Arch: by which alſo it is demonſtrable, that the Side of a Triangle hath the ſame Proportion as the ſines of their oppoſite Angles. We furthermore make uſe of this Propoſition to find the Excentricity of the Circle which the Sun deſcribeth in a Year.

PRO-

PROPOSITION V.

THEOREM.

CIrcles which cut each other have not the ſame Center.

The Circles A B C, A D C, which cut each other in the Points A and C, have not the ſame Center: for if they had the ſame Center E, the Lines E A, E D, would then be equal (*by the definition of a Circle*,) as alſo the Lines E A, E B: thence the Lines E D, E B, would be equal; which is impoſſible, the one being a part of the other.

PROPOSITION VI.

THEOREM.

IF two Circles inwardly touch one the other, they have not one and the ſame Center.

The Circles B D, B C, which touch inwardly one the other, in the Point B, have not the ſame Center: for if the Point A was the Center of both Circles, the Lines A B, A D, and A B, A C, would be equal (*by the definition of a Cir-*

Circle) and ſo A D, A C, would be equal; which is impoſſible, the one being a Part of the other.

PROPOSITION VII.

THEOREM.

IF in a Circle ſeveral Lines be drawn from a Point which is not the Center, unto the Circumference; firſt, that which paſſeth through the Center ſhall be greateſt; ſecondly, the leaſt ſhall be the Remainder of the ſame Line; thirdly, the neareſt Line to the greateſt is greater than any Line which is farther from it; fourthly, there cannot be drawn more than two equal Lines.

Let there be drawn ſeveral Lines from the Point A, which is not the Center of the Circle, to the Circumference; and let the Line A C paſs through the Center B: I demonſtrate that it ſhall be the greateſt or longeſt, that it is greater than A F. Draw F B.

Demonſtration. The ſides A B, B F, of the Triangle A B F, are greater than the ſide A F, (*by the 20th of the I.*) Now, B F, B C, are equal (*by the definition of a Circle*) therefore A B, B C, that

that is to say, A C, is greater than A F. Moreover, in the second place, I say, that A D is the shortest; for Example, shorter than A E. Draw E B.

Demonstration. The Sides E A, A B, of the Triangle A B E, are greater than B E, which is equal to B D: thence E A, A B, are greater than B D; and taking away A B, which is common, A E shall be greater than A D. Moreover, A F, which is nearer to A C than A E, is greater than A E.

Demonstration. The Triangles F B A, E B A, have the Sides B F, B E, equal, and B A common to both: the Angle A B F is greater than the Angle A B E; thence (*by the* 24th *of the first*) A F is greater than A E.

In fine, I say that there cannot be drawn but two equal Lines from the Point A, to the Circumference, let the Angles A B E, A B G, be equal, and let be drawn the Line A E, A G.

Demonstration. The Triangles A B G, A B E, have their Bases A E, A G, equal; and all the Lines which may be drawn on either side of these, shall be either nearer to A C than the Lines A E, A G, or farther: and so they shall be either shorter or longer than A E,

A E, A G. Therefore, there cannot be drawn more than two equal Lines.

USE.

THeodosius *doth very well make use of this Proposition, to prove, that if from a Point in the Superficies of a Sphere, which is not the Pole of a Circle, there be drawn several Arks of great Circles unto the Circumference of a Circle: that which passeth through the Pole of that Circle to which the great Circles are drawn, is greatest. For Example, if from the Pole of the World, which is not the Pole of the Horizon (for the Zenith is the Pole thereof) there be drawn several Arks of great Circles unto the Circumference of the Horizon: the greatest Ark of all shall be that part of the Meridian which passeth through the Zenith. This Proposition is also made use of, to prove, that the Sun being in Apoge, is farthest distant from the Earth.*

PRO-

PROPOSITION VIII.

THEOREM.

IF from a Point without a Circle, be drawn ſeveral Lines to its Circumference; firſt, of all thoſe which are drawn to its concave Circumference, the greateſt paſſeth through the Center; ſecondly, thoſe nearer thereto are greater than thoſe which are farther off; thirdly, amongſt the Lines which fall on the convex Circumference, the leaſt being continued, paſſeth through the Center; fourthly, the nearer thereto are leaſt; fifthly, there cannot be drawn but only two equal, whether they be drawn to the convex Circumference, or that they fall on the concave.

Let there be drawn from the Point A, ſeveral Lines to the Circumference of the Circle G C, D E. In the firſt place the Line A C, which paſſeth through the Center B, is the greateſt of all thoſe which fall on the concave Circumference: for Example, it is greater than A D. Draw the Line B D.

Demonſtration. In the Triangle ABD the Sides A B, B D, are greater than the Side

Side A D; now the Sides A B, B C, are equal to A B, B D; thence A B, B C, or A C, is greater than A D.

2. A D is greater than A E.

Demonſtration. The Triangles A B D, A B E, have the Side A B common, and the Sides B E, B D, equal; and the Angle A B D, is greater than the Angle A B E; thence (*by the* 24th. *of the* 1ſt.) the Baſe A D is greater than the Baſe A E.

3. A F, which being prolonged paſſeth through the Center, is the leaſt of thoſe Lines which are drawn to the convex Circumference L F I K, For example, it is leſs than A I. Draw I B.

Demonſtration. The Sides A I, I B are greater than A B, (*by the* 20th. *of the* 1ſt.) wherefore taking away the equal Lines B I, B F, A F ſhall be leſs than A I.

4. A I is leſs than A K. Draw the Line B K

Demonſtration. In the Triangles A I B, A K B, the ſides A K, K B, are greater than the Sides A I, I B, (*by the* 21th. *of the* 1ſt.) wherefore taking away the equal ſides, B K, B I, there remains A I leſs than A K.

5. Ther

5. There can be drawn but two equal Lines; make the Angles A B L, A B K, equal.

Demonstration. The Triangles A B L, A B K, shall have their Bases A L, A K, equal (*by the* 4th. *of the* 1st.) but there cannot be drawn any other, which will not be either nearer or farther from A F, and which will not be either greater or lesser than A K, A L.

PROPOSITION IX.

THEOREM.

THe Point from whence may be drawn to the Circumference of a Circle, three equal Lines, is its Center.

If that Point were not the Center of the Circle, there could be drawn therefrom only two equal Lines (*by the* 7th.)

PROPOSITION X.

THEOREM.

TWo Circles cut each other only in two Points.

If the two Circles A B D, A B F D, did cut each other in three Points, A, B, D;

B, D; ſeek *(by the* 2^d.*)* the Center C of the Circles A E, B D, and draw the Lines C A, C B, C D.

Demonſtration. The Lines A C, B C, D C, drawn from the Center C, to the Circumference of the Circle A E, B D, are equal: Now the ſame Lines are alſo drawn to the Circumference of the Circle A B, F B: thence *(by the* 9th.*)* the Point C ſhall be the Center of the Circle A B F D. So two Circles which cut each other ſhall have the ſame Center; which is contrary to the fifth Propoſition.

PROPOSITION XI.

THEOREM.

IF two Circles inwardly touch one the other, the Line drawn through both Centers, paſſeth alſo through the Point, where they touch one the other.

If the two Circles E A B, E F G, inwardly touch one the other, in the Point E, the Line drawn through both Centers ſhall paſs through the Point E. Now if the Point D be the Center of the leſſer Circle, and C that of the greater, in ſuch manner, that the

Line

Line C D passeth not through the Point E, draw the Lines C E, D E.

Demonstration. The Lines D E, D G, drawn from the Center D of the lesser Circle, are equal; and adding the Line D C; the Lines E D, D C, should be equal to C G. Now E D, D C, are greater than E C (*by the* 20th *of the* 1st.) so that C G is greater than C E, and notwithstanding C being the Center of the great Circle; C E, C B, are equal; thence C G would be greater than C B, which is impossible.

PROPOSITION XII.

THEOREM.

IF Two Circles touch one the other outwardly, the Line drawn through their Centers passeth through the Point where they touch one the other.

If the Line A B drawn from the Center A, to the Center B, passeth not through the Point C, where the Circles touch one the other; draw the Lines A C, B C.

Demonſtration. In the Triangle ACB, the Side AB would be greater than AC, BC; for BC, BE, are equal, as well as AD, AC; which would be contrary to the 20*th* Propoſition of the 1*ſt*.

PROPOSITION XIII.

THEOREM.

TWo Circles touch one the other only in one Point.

In the firſt place, if Two Circles touch one the other inwardly, they ſhall touch one the other only in one Point C, in the Line BAC, which paſſeth through their Centers A and B. Now if it be ſuppoſed that they touch one the other again in the Point D; draw the Lines AD, DB.

Demonſtration. The Lines AD, AC, being drawn from the Center of the leſſer Circle, are equal, and adding to both AB the Lines BA AC, BA, AD ſhould be equal: Now BC, BD being drawn from the Center of the greater Circle, ſhould be alſo equal; thence the Sides BA, AD, ſhould be equal to the Side BD; which

which is contrary to the 20th of the 1st.

Secondly, if the Two Circles touch outwardly; drawing the Line A B from one Center to the other, it ſhall paſs through the Point C, where the Circles touch (*by the* 12th.) Now if you ſay they alſo touch in the Point D; having drawn the Lines A D, B D; the Lines B D, B C; A D, A C, being equal, the Two Sides of a Triangle taken together, ſhould then be equal to the Third, which is contrary to the 20th of the 1ſt,

USE.

These Four Propoſitions are very clear, they are notwithſtanding neceſſary in Aſtronomy, when we make uſe of Epicycles *to explain the motion of the Planets.*

PROPOSITION XIV.

THEOREM.

Equal Right Lines drawn in a Circle, are equally diſtant from the Center and Right Lines, which are equally diſtant from the Center, are equal.

If the Lines A B, C D, are equal; I

demonſtrate that the Perpendicular E F, E G, drawn from the Center are alſo equal. Draw the Lines E A, E C.

Demonſtration. The Perpendiculars E F, E G, divideth the Lines A B, C D, in the middle in F and G (*by the* 3d:) So A F, C G, are equal, the Angles F and G are Right; thence (*by the* 47th *of the* 1ſt) the Square of E A is equal to the Squares of E F, F A; as the Square of E C is equal to the Squares of C G, E G: Now the Squares of E A, E C, are equal, the Lines E A, E C, being equal: Therefore the Squares of E F, F A, are equal to the Squares of E G, G C; and taking away the equal Squares A F, C G; there remains the equal Squares E G, E F; ſo then the Lines E G, E F, which are the diſtances are equal.

If you ſuppoſe that the diſtances, or perpendiculars E G, E F, are equal; I would demonſtrate after the ſame manner that the Squares of E F, F A, are equal to the Squares of E G, G C, and taking away the equal Squares of E F, E G, there ſhall remain the equal Squares A F, C G; ſo then the Lines A F, C G, and their double A B, C D, are equal.

PRO-

PROPOSITION XV.

THEOREM.

THe Diameter is the greateſt Line drawn in a Circle, and thoſe neareſt the Center are greateſt.

The Diameter A B is the greateſt of the Lines that are drawn in the Circle; for example, it is greater than C D. Draw the Lines E C, E D.

Demonſtration. In the Triangle C E D, the Two Sides E C, E D, are greater than the Side C D (*by the* 20th *of the* 1ſt.) Now A E, E B, or A B is equal to E C, E D: Thence the Diameter A B is greater than C D.

Secondly, Let the Line G I, be farther diſtant from the Center than the Line C D; that is to ſay, let the perpendicular E H be greater than the Perpendicular E F: I ſay that C D is greater than G I. Draw the Lines E C, E G.

Demonſtration. The Squares C F, F E, (*by the* 47th *of the* 1ſt.) are equal to the Square of C E: Now C E is equal to E G, and the Square of E G is equal to the Squares of G H, H E:

By consequence the Squares of C F, F E, are equal to the Squares of G H, H E; and taking away on the one Side the Square of H E, and on the other the Square of F E, which is less; the the Square of C F shall be greater than the Square of G H. Thence the Line C F shall be greater than G H; and the whole Line C D, the double of C F shall be greater than G I, the double of G H.

U S E.

THeodosius *maketh use of those Two last Propositions to Demonstrate, that in the Sphere, the least Circles are farthest distant from the Center. We make use thereof in* Astrolabes. *Moreover to those Propositions might be brought the question of Mechanicks, proposed by* Aristotle, *which certifies that the Rowers which are in the middle of a Gulley, has a greater strength than those which are a stern or a head, because that the sides of the Gulley being rounded, have the Oars which be there placed longest. Also the Demonstration of the Rainbow in the Heavens supposeth this Proposition.*

PRO-

PROPOSITION XVI.

THEOREM.

THe Perpendicular Line drawn to the extream part of the Diameter is wholly without the Circle, and toucheth the ſame. All other Lines drawn between it, and the Circumference of the Circle cut it, and go within the ſame.

Draw by the Point A, which is the extremity of the Diameter A B, the Perpendicular A C; I ſay firſt, that all other Points of the ſame Line, as the Point C, are without the Circle. Draw the Line D C.

Demonſtration. Becauſe the Angle D A C of the Triangle D A C, is a Right Angle; D C A ſhall be acute, and (*by the* 19th *of the* 1ſt,) the Side D C ſhall be greater than the Side D A; thence the Line D C paſſeth beyond the Circumference of the Circle.

I farther add that the Line C A toucheth the Circle, becauſe in the meeting thereof in the Point A, it cutteth it not, but hath all its other Points without the Circle.

I say also that there cannot be drawn any other Line from the Point A, underneath C A, which shall not cut the Circle; as for example, the Line E A. Draw the Perpendicular D I.

Demonstration. Seeing the Angle I A B is Acute, the Perpendicular drawn from the Point D shall be on the same Side with the Angle I A B (*by the* 17th *of the* 1st.) Let it be D I; the Angle D I A is Right, and the Angle I A D Acute; A D shall be greater than D I; thence the Line D I, doth not reach the Circumference, and the Point I is within the Circle.

PROPOSITION XVII.

PROBLEM.

From a Point taken without the Circle, to draw a Line to touch the same.

To draw a touch Line, from the Point A to the Circle B D; draw the Line A C to its Center; and at the Point B, the Perpendicular B E; which cutteth in E a Circle described on the Center C, passing through the Point A. Draw also the Line C E. I say that the Line

Line A D toucheth the Circle in the Point D.

Demonſtration. The Triangle E B C, A D C, have the ſame Angle C; and the Sides C D, C B; C A C E, equal, *by the Definition of a Circle*: ſo they are equal in every reſpect (*by the* 4th *of the* 1ſt:) and the Angles C B E, C D A are equal: Now C B A is Right; and (*by the* 16th,) the Line A D ſhall touch the Circle.

PROPOSITION XVIII.

THEOREM.

THe Line drawn from the Center of a Circle to the Point where a ſtreight Line toucheth the ſame, is Perpendicular to the ſame Line.

If there be drawn the Line C D, from the Center C, to the Point of touching D: C D ſhall be Perpendicular to A B. For if it be not; draw B C Perpendicular to A B.

Demonſtration. Since we would have it that the Line C B be Perpendicular; the Angle B ſhall be Right, and C D B Acute, (*by the* 32d *of the* 1ſt.) Thence the Line C B, oppoſite

to the leſſer Angle, ſhall be leſſer than CD; which is impoſſible, ſeeing CF is equal to CD.

PROPOSITION XIX.

THEOREM.

THe Perpendicular drawn to a touch Line at the Point of touching Paſſeth through the Center.

Let the Line A B touch the Circle in the Point D; and let the Line D C be Perpendicular to A B: I ſay that D C paſſeth through the Center. For if it paſſeth not through it, drawing from the Center to the Point D a Line; it would be Perpendicular to A B: and ſo Two Lines Perpendicular to the ſame, would be drawn to the ſame Point D; which cannot be.

USE.

THe uſe of touch Lines is very common in Trigonometry, which hath obliged us to make a Table thereof, it ſerveth us to meaſure all ſorts of Triangles, even Spherical. We have in Opticks ſome Propoſitions founded on thoſe touch

ouch Lines, as when we are to determine the part of a Globe which is enlightned. The Theorie of the Phaſes of the Moon is eſtabliſhed on the ſame Doctrine, as alſo that Celebrated Problem by which Hipparchus *findeth the diſtance of the Sun, by the difference of the true and apparent Quadratures. In the Gnomonicks, the Italian and Babylonian hours, are often Tangent Lines. We meaſure the Earth by a Line which toucheth its Surface. And we take in the Art of Navigation, a touch Line to be our Horizon.*

PROPOSITION XX.

THEOREM.

THe Angle of the Center is double to an Angle of the Circumference, which hath the ſame Arch for Baſe.

If the Angle A B C which is in the Center, and the Angle A D C which is in the Circumference, have the ſame Arch A C for Baſe; the firſt ſhall be double to the ſecond, this Propoſition hath three Caſes; the firſt is when the Line A B D paſſeth through the Center B.

Demonſtration.

Demonſtration. The Angle A B C is exteriour in reſpect of the Triangle B D C: Thence (*by the* 32^d *of the* 1ſt.) it ſhall be equal to the Two Angles D, and C; which being equal (*by the* 5th *of the* 1ſt,) becauſe the Sides B C, B D, are equal; the Angle A B C ſhall be double of each.

The Second caſe is, when an Angle encloſeth the other, and the Lines making the ſame Angles, not meeting each other, as you ſee in the ſecond figure; the Angle B I D is in the Center, and the Angle B A D is at the Circumference. Draw the Line A I C through the Center.

Demonſtration. The Angle B I C is double to the Angle B A C; and C I D, double to the Angle C A D, (*by the preceding caſe*:) Therefore the Angle B I D ſhall be double to the Angle B A D.

USE.

THere is given ordinarily a practical way to deſcribe a Horizontal Dial, by a ſingle opening of the Compaſs, which is grounded in part on this Propoſition. Secondly, when we would determine the Apogæon of the Sun, and the excentricity of

of his Circle, by Three obſervations; we ſuppoſe that the Angle at the Center is double to the Angle at the Circumference. Ptolomy makes often uſe of this Propoſition, to determine as well the excentricity of the Sun, as the Moon's epicycle. The firſt Propoſition of the Third Book of Trigonometry is grounded on this.

PROPOSITION XXI.

THEOREM.

THe Angles which are in the ſame Segment of a Circle, or that have the ſame Arth for Baſe, are equal.

If the Angles B A C, B D C, are in the ſame Segment of a Circle, greater than a Semicircle; they ſhall be equal. Draw the Lines B I, C I.

Demonſtration. The Angles A and D are each of them half of the Angle B I C, by the preceding Propoſition; therefore they are equal. They have alſo the ſame Arch B C for Baſe.

Secondly, let the Angles A and D be in a Segment B A C, leſs than a ſemicircle; they ſhall notwithſtanding be equal.

Demonſtration.

Demonſtration. The Angles of the Triangle A B E are equal to all the Angles of the Triangle D E C: The Angles E C D, A B E, are equal (*by the preceding caſe*;) ſince they are in the ſame Segment A B C D, greater than a Semi-circle: the Angles in E are likewiſe equal (*by the* 15th. *of the* 1ſt.) therefore the Angles A and D ſhall be equal, which Angles have alſo the ſame Arch B F C, for Baſe.

USE.

Prop. XXI. *IT is proved in* Opticks, *that the Line* B C *ſhall appear equal, being ſeen from* A *and* D; *ſince it always appeareth under equal Angles.*

We make uſe of this Propoſition to deſcribe a great Circle, without having its Center; For Example, when we would give a Spherical figure, to Braſs Cauldrons to the end we may work thereon; and to polliſh Proſpective or Teleſcope Glaſſes. For having made in Iron an Angle B A C *equal to that which the Segment* A B C *contains; and having put in the Points* B, *and* C, *two ſmall pins of Iron, if the Triangle* B A C *be made to move after ſuch a manner, that the Side* A B *may always touch the Pin* B;

B; and the Side AC, the Pin C: the Point A shall be always in the Circumference of the Circle ABCD. This way of describing a Circle may also serve to make large Astrolabes.

PROPOSITION XXII.

THEOREM.

Quadrilateral figures described in a Circle, have their opposite Angles equal to Two Right.

Let a Quadrilateral or four sided figure, be described in a Circle; in such manner that all its Angles touch the Circumference of the Circle ABCD: I say that its opposite Angles BAD, BCD, are equal to two Right. Draw the Diagonals AC, BD.

Demonstration. All the Angles of the Triangle BAD are equal to Two Right. In the place of its Angle ABD, put the Angle ACD, which is equal thereto (*by the* 21st.) as being in the same Segment ABCD; and in the place of its Angle ADB, put the Angle ACB, which is in the same Segment of the Circle BCDA. So then the Angle BAD, and the Angles ACD,

A C D, A C B, that is to say the whole Angle B C D, is equal to Two Right.

USE.

PTolomy *maketh use of this Proposition, to make the Tables of Chords or Subtendents. I have also made use thereof in Trigonometry, in the Third Book, to prove that the sides of an Obtuse Angled Triangle, hath the same reason amongst themselves, as the Sines of their opposite Angles.*

PROPOSITION XXIII.

THEOREM.

TWo like Segments of a Circle described on the same Line, are equal.

I call like Segments of a Circle, those which contain equal Angles; and I say that if they be described on the same Line A B, they shall fall one on the other, and shall not surpass each other in any part. For if they did surpass each other, as doth the Segment A C B, the Segment A D B; they would not be like. And to demonstrate it, draw

draw the Lines ADC, DB, and BC.

Demonſtration. The Angle ADB is exteriour in reſpect of the Triangle BDC: Thence (*by the* 21th. *of the* 1ſt.) it is greater than the Angle ACB, and by conſequence, the Segments ADB, ACB, containeth unequal Angles, which I call unlike.

PROPOSITION XXIV.

THEOREM.

TWo like Segments of Circles deſcribed on equal Lines, are equal.

If the Segments of Circles AEB, CFD, are like, and if the Lines AB, CD, are equal, they ſhall be equal.

Demonſtration. Let it be imagined that the Line CD be placed on AB, they ſhall not ſurpaſs each other, ſeeing they are ſuppoſed equal; and then the Segments AEB, CED, ſhall be deſcribed on the ſame Line; and they ſhall then be equal by the preceding Propoſition.

USE.

USE.

Use 24. *Curved Lined Figures are often reduced to Right Lined by this Proposition. As if one should describe Two like Segments of Circles, A E C, A D B, on the equal sides A B, A C, of the Triangle A B C: It is evident that Transposing the Segment A E C, on A D B; the Triangle A B C is equal to the figure A D B C E A.*

PROPOSITION XXV.

PROBLEM.

TO compleat a Circle whereof we have but a part.

There is given the Arch A B C, and we would compleat the Circle. There needeth but to find its Center. Draw the Lines A B, B C, and having Divided them in the middle in D and E, draw their Perpendiculars D I, E I, which shall meet each other in the Point I, the Center of the Circle.

Demonstration.

Demonstration. The Center is in the Line D I (*by the Coroll. of the* 1st.) It is also in E I; it is then in the Point I.

USE.

THis Proposition cometh very frequently in use; it might be propounded another way; as to inscribe a Triangle in a Circle; or to make a Circle pass through three given points, provided they be not placed in a streight Line. Let be proposed the Points A, B, C; put the Point of the Compass in C, and at what opening soever describe Two Arks F and E. Transport the foot of the Compass to B; and with the same opening, describe Two Arks to cut the former in E and F. Describe on B as Center, at what opening soever, the Arches H and G; and at the same opening of the Compass, describe on the Center A Two Arks, to cut the same in G and H. Draw the Lines F E, and G H, which will cut each other in the Point D the Center of the Circle. The Demonstration is evident enough; for if you had drawn the Lines A B, B C, you would have divided them equally, and Perpendicularly by so doing. This is very necessary to describe Astrolabes, *and to* Use 25.

compleat

compleat Circles, of which we have but a part. That Aſtronomical Propoſition which teacheth to find the Apogeum, and the excentricity of the Suns Circle, require this Propoſition. We often make uſe thereof in the Treatiſe of cutting of ſtones.

PROPOSITION XXVI.

THEOREM.

THe equal Angles which are at the Center, or at the Circumference of equal Circles, have for Baſe equal Arks.

If the equal Angles D and I are in the Center of equal Circles A B C, E F G; the Arks B C, F G, ſhall be equal. For if the Ark B C was greater or leſſer than the Ark F G: ſeeing that the Arks are the meaſure of the Angles; the Angle D would be greater or leſſer than the Angle I.

And if the equal Angles A and E be in the Circumference of the equal Circles; the Angles D and I which are the double of the Angles A and E, being alſo equal; the Arks B C, F G, ſhall be alſo equal.

PRO-

PROPOSITION XXVII.

THEOREM.

THE Angles which are either in the Center or in the Circumference of equal Circles, and which hath equal Arks for Baſe, are alſo equal.

If the Angles D and I are in the Centers of equal Circles; and if they have for Baſe equal Arcks B C, F G, they ſhall be equal, becauſe that their meaſures B C, F G, are equal; and if the Angles A and E be in the Circumference of equal Circles, have for Baſe equal Arks B C, E G, the Angles in the Center ſhall be equal; and they being their halfs (*by the* 20th.) ſhall be alſo equal.

PROPOSITION XXVIII.

THEOREM.

EQual Lines in equal Circles, correſpond to equal Arks.

If the Line B C, E F, are applyed in equal Circles, A B C, D E F; they ſhall be Chords of equal Arks, B C, E F.

Draw

Draw the Lines A B, A C, D E, E F.

Demonſtration. In the Triangles A B C, D E F, the Sides A B, A C, D E, E F, are equal, being the Semi-Diameters of equal Circles; the Baſes B C, E F, are ſuppoſed equal: thence (*by the* 8th *of the* 1ſt.) the Angles A and D ſhall be equal; and (*by the* 16th.) the Arks B C, E F, ſhall be alſo equal.

PROPOSITION XXIX.

THEOREM.

Lines which ſubtend equal Arcks, in equal Circles, are equal.

If the Lines B C, E F, ſubtend equal Arks B C, E F, in equal Circles; thoſe Lines are equal.

Demonſtration. The Arks B C, E F, are equal, and parts of equal Circles: therefore (*by the* 27th,) the Angles A and D ſhall be equal. So then in the Triangles C A B, E D F, the Sides A B, A C, D E, D F, being equal, as alſo the Angles A and D; the Baſes B C, E F, ſhall be equal (*by the* 4th. *of the* 1ſt.)

USE.

USE.

THeodosius *demonstrateth by the* 28th, *and* 29th, *that the Arks of the Circles of the* Italian *and* Babylonian *hours comprehended between Two Parallels are equal. We demonstrate also after the same manner, that the Arks of Circles of Astronomical hours comprehended between Two Parallels to the Equator, are equal; these Propositions come almost continually in use in spherical Trigonometry, as also in Gnomonicks.*

PROPOSITION XXX.

PROBLEM.

TO divide an Ark of a Circle into Two equal parts.

It is proposed to Divide the Ark AEB into Two equal parts, put the Foot of the Compass in the Point A, make Two Arks F and G; then transporting the Compass without opening or shutting it, to the Point B: describe two Arks cutting the former in F and G; the Line GF, will cut the Ark AB

A B equally in the Point E. Draw the Line A B.

Demonſtration. You divide the Line A B equally by the conſtruction, for imagine the Lines A F, B F, A G, B G, which I have not drawn, leſt I ſhould imbroil the figure; the Triangles F G A, F G B, have all their Sides equal; ſo then (*by the* 8th. *of the* 1ſt.) the Angles A F D, B F D, are equal. Moreover the Triangles D F A, D F B, have the Sides D F, common; the Sides A F, B F, equal, and the Angles D F A, D F B, equal; whence (*by the* 4th *of the* 1ſt,) the Baſes A D, D B, are equal; and the Angles A D F, B D F, are equal. We have then divided the Line A B equally and perpendicularly in the Point D. So then (*by the* 1ſt,) the Center of the Circle is in the Line E G. Let it be the Point C, and let be drawn the Lines C A, C B; all the Sides of the Triangles A C D, B C D, are equal: Thence the Angles A C D, B C D, are equal (*by the* 8th *of the* 1ſt;) and (*by the* 27th,) the Arks A E, E B, are equal.

Q S E.

USE.

As we have often need to divide an Ark in the middle, the practice of this Proposition is very ordinarily in use, it is by this means we divide the Mariners Compass into 32 Rumbs: for having drawn Two Diameters, which cut each other at Right Angles; we divide the Circle in Four, and sub-dividing each quarter in the middle, we have Eight parts, and sub-dividing each part twice, we come to Thirty Two parts. We have also occasion of the same practice, to divide a Semi-circle into 180 degrees; and because for the performing the same Division throughout, we are obliged to divide an Ark into Three, all the Ancient Geometricians have endeavoured to find a method to divide an Angle or an Ark into Three equal parts, but it is not yet found.

PROPOSITION XXXI.

THEOREM.

The Angle which is in a Semi-circle is Right, that which is comprehended in a greater Segment, is Acute; and that in a lesser Segment, is Obtuse.

If the Angle B A C be in a Semicircle: I demonſtrate that it is Right. Draw the Line D A.

Demonſtration. The Angle A D B exteriour in reſpect of the Triangle D A C is equal (*by the* 32*d. of the* 1*ſt.*) to the Two Interiours D A C, D C A; and thoſe being equal (*by the* 5*th. of the* 1*ſt.*) ſeeing the Sides D A, D C, are equal; it ſhall be double to the Angle D A C. In like manner the Angle A D C is double to the Angle D A B: therefore the Two Angles A D B, A D C, which are equal to Two Right, are double to the Angle B A C, and by conſequence the Angle B A C is a Right Angle.

Secondly, the Angle A E C which is in the Segment A E C, is obtuſe; for in the Quadrilateral A B C E, the Oppoſite Angles E and B, are equal to Two Right (*by the* 22*d.*) the Angle B is Acute; therefore the Angle E ſhall be Obtuſe.

Thirdly, the Angle B which is in the Segment A B C, greater than a Semicircle, is Acute; ſeeing that in the Triangle A B C, the Angle B A C is a Right Angle.

USE.

THE Workmen have drawn from this Propoſition the way of trying if their Squares be exact; for having drawn a Semi-circle B A D, they apply the Point A of their Square B A D, on the Circumference of the Circle, and one of its Sides A B, on the Point B of the Diameter; the other Side A D muſt touch the other Point D, which is the other end of the Diameter. Uſe 31.

Ptolomy *makes uſe of this Propoſition to make the Table of Subtendants or Chords, of which he hath occaſion in Trigonometry.*

We have alſo a practical way to erect a perpendicular on the end of a Line, which is founded on this Propoſition. For Example, to erect a Perpendicular from the Point A of the Line A B, I put the Foot of the Compaſs on the Point C, taken at diſcretion; and extending the other to A, I deſcribe a Circle which may cut the Line A B in the Point B. I draw the Line B C D. It is evident that the Line D A ſhall be perpendicular to the Line A B; ſeeing the Angle B A D is in the Semi-circle. Uſe 31.

PROPOSITION XXXII.

THEOREM.

THE Line which cutteth the Circle at the Point of touching, maketh with the touch Line the Angles equal to those of the Alternate Segments.

Let the Line B D cut the Circle in the Point B, which is the Point where the Line A B doth touch the same: I say that the Angle C B D which the Line B D Comprehendeth with the touch Line A B C, is equal to the Angle E, which is that of the Alternate Segment B E D; and that the Angle A B D, is equal to the Angle F, of the Segment B F D.

In the first place, if the Line passeth through the Center, as doth the Line B E: It would make with the touch Line A B, two Right Angles (*by the 17th.*) and the Angle of the Semi-circles would be also Right (*by the preceding.*) So the Proposition would be true.

If the Line passeth not through the Center, as doth the Line B D: Draw the Line B E through the Center, and joyn the Line D E:

Demonstration.

Demonſtration. The Line B E maketh two Right Angles with the touch Line, and all the Angles of the Triangle B D E are equal to Two Right (*by the 32d. of the 1ſt.*) So taking away the Right Angles ABE, and D, which is in a Semi-circle, and taking again away the Angle E B D, which is common to both; the Angle C B D ſhall be equal to the Angle B E D.

Thirdly, the Angle A B D is equal to the Angle F, becauſe in the Quadrilateral B F D E, which is inſcribed in a Circle, the oppoſite Angles E, and F, are equal to two Right (*by the 22d.*) the Angles A B D, D B C, are alſo equal to Two Right (*by the 13th. of the 1ſt.*) and the Angle D B C, and E, are equal, as juſt now I did demonſtrate; therefore the Angles A B D, and B F D, are equal.

U S E.

THis Propoſition is neceſſary for that which followeth.

PROPOSITION XXXIII.

PROBLEM.

TO deſcribe upon a Line, a Segment of a Circle, which ſhall contain a given Angle.

It is propoſed to deſcribe on the Line A B, a Segment of a Circle to contain the Angle C. Make the Angle B A D equal to the Angle C; and draw to A D the Perpendicular A E. Make alſo the Angle A B F, equal to the Angle B A F; and laſtly deſcribe a Circle on the Point F as Center, at the opening B F, or F A, the Segment B E A containeth an Angle equal to the Angle C.

Demonſtration. The Angles B A F, A B F, being equal, the Lines F A, F B, are equal (*by the 6th.*) and the Circle which is deſcribed on the Center F paſſeth through A and B: Now the Angle D A E being Right, the Line D A toucheth the Circle in the Point A, (*by the 16th.*) therefore the Angle which the Segment B E A comprehendeth as the Angle E, is equal to the Angle D A B; that is to ſay to the Angle C. But

But if the Angle was obtuſe; we muſt take the Acute Angle which is its Complement to 180 degrees.

PROPOSITION XXXIV.

PROBLEM.

A Circle being given, to cut there-from a Segment to contain an aſſigned Angle.

To cut from the Circle BCE, a Segment to contain the Angle A. Draw (*by the* 17*th.*) the touch Line BD; and make the Angle DBC equal to the Angle A. It is evident (*by the* 32*d.*) that the Segment BEC contains an Angle equal to DBC, and by conſequence to the Angle A.

USE.

I Have made uſe of this Propoſition to find Geometrically the excentricity of the Annual Circle of the Sun, and his Apogeon, Three obſervations being given. It is alſo made uſe of in Opticks, Two unequal Lines being propoſed, to find a Point where they ſhall appear equal, or under equal

Angles; making on each, Segments which may contain equal Angles.

PROPOSITION XXXV.

THEOREM.

IF Two Lines cut each other in a Circle, the Rectangle comprehended under the parts of the one, is equal to the Rectangle comprehended under the parts of the other.

In the first place, if Two Lines cut each other in the Center of the Circle, they shall be equal and divided equally, so then it is evident, that the Rectangle comprehended under the parts of the one, is equal to the Rectangle comprehended under the parts of the other.

Secondly, let one of the Lines pass through the Center F; as A C, and divide the Line B D in two equally in the Point E: I say that the Rectangle comprehended under A E, E C, is equal to the Rectangle comprehended under B E, E D, that is to say to the Square of B E. The Line A C is perpendicular to B D (*by the Third.*)

Demonstration.

Demonſtration. Seeing that the Line A C is divided equally in F, and unequally in E; the Rectangle comprehended under A E, E C, with the Square of F E, is equal to the Square of F C, or F B, (*by the 5th. of the 2d.*) Now the Angle E being Right, the Square of F B is equal to the Squares of B E, E F; therefore the Rectangle comprehended under A E, E C, with the Square of E F, is equal to the Squares of B E, E F; and taking away the Square of E F, there remains that the Square of B E, is equal to the Rectangle under B E, E D.

Thirdly, let the Line AB paſs through the Center F, and let it divide the Line C D unequally in the Point E: draw F G perpendicularly to C D; and (*by the 3d.*) the Lines C G, G D, ſhall be equal.

Demonſtration. Seeing the Line A B is divided equally in F, and unequally in E; the Rectangle comprehended under A E, E B, with the Square of E F, is equal to the Square of B F, or F C, (*by the 5th. of the 2d.*) In the place of E F, put the Squares of F G, G E, which is equal thereto, (*by the 47th. of the 1ſt.*) In like manner the

Line C D being equally divided in G, and unequally in E; the Rectangle C E D, with the Square of G E, ſhall be equal to the Square of G C. Add the Square of G F; the Rectangle of C E, E D, with the Squares of G E, F G, ſhall be equal to the Squares of G C, G F; that is to ſay (*by the 47th. of the 1ſt.*) to the Square of C F. Therefore the Rectangle A E B, with the Squares of G E, G F; and the Rectangle of C E, E D, with the ſame Squares are equal; and by conſequence taking away the ſame Squares, the Rectangle A E B is equal to the Rectangle C F D.

Fourthly, let the Lines C D, H I, cut each other in the Point E, ſo that neither of them paſs through the Center. I ſay that the Rectangle C E D is equal to the Rectangle H E I. For drawing the Line A F B, the Rectangles C E D, H E I, are equal to the Rectangle A E B, (*by the preceding caſe.*) therefore they are equal.

USE.

ONe might by this Propoſition have a practical way to find the Fourth proportional to Three given Lines, or the Third proportional to Two Lines.

PROPOSITION XXXVI.

THEOREM.

IF from a Point taken without a Circle, one draw a touch Line, and another Line to cut the Circle, the Square of the Tangent Line ſhall be equal to the Rectangle comprehended under the whole Secant, and under the exteriour Line.

Let from the Point A, taken without the Circle, be drawn a Line A B, to touch the ſame in B; another A C, or A H, to cut the Circle. The Square of A B ſhall be equal to the Rectangle comprehended under A C, A O; as alſo to the Rectangle comprehended under AH, A F. If the Secant paſs through the Center, as A C; draw the Line E B.

Demonſtration. Seeing the Line O C, is divided in the middle in E, and that thereto is added the Line A O; the Rectangle

Rectangle comprehended under AO, AC, with the Square of OE, or EB, ſhall be equal to the Square of AE, (*by the 6th. of the 2d.*) Now the Line AB toucheth the Circle in the Point B: ſo (*by the 17th.*) the Angle B is Right; and (*by the 47th. of the 1ſt.*) the Square of AE is equal to the Squares of AB, EB. Therefore the Rectangle under AC, AO, with the Square of EB, is equal to the Squares of AB, EB: and taking away the Squares of EB; the Rectangle under AC, AO, ſhall be equal to the Square of AB.

Secondly, let the Secant AH not paſs through the Center. Draw the Line AH, the perpendicular EG, which ſhall divide in the middle in G, the Line FH: draw alſo the Line EF.

Demonſtration. The Line FH being divided equally in the point G; and the Line AF being added thereto; the Rectangle comprehended under AH, AF, with the Square of FG, ſhall be equal to the Square of AG. Add to both the Square of EG; the Rectangle AH, AF, with the Square of FG, GE, that is to ſay (*by the 47th. of the 1ſt.*) with the Square of FE or EB, ſhall be equal to the Square of AG, GE,

G E, that is to say (*by the 47th. of the 1st.*) to the Square of A E. Moreover, the Square of A E, (*by the same,*) is equal to the Square of E B, A B: therefore the Rectangle compprehended under A H, A F, with the Square of B E, is equal to the Squares of B E, A B, and taking from both the Square of B E, the Rectangle comprehended under A H, A F, shall be equal to the Square of A B.

Coroll. 1. If you draw several Secants A C, A H; the Rectangles A C, A O; and A H, A F, shall be equal amongst themselves, seeing that the one and the other are equal to the Square of A B.

Coroll. 2. If there be drawn Two Tangents A B, A I, they shall be equal because their Squares are equal to the same Rectangle A C, A O; and by consequence amongst themselves; as also the Lines.

PROPOSITION XXXVII.

THEOREM.

IF the Rectangle comprehended under the Secant, and under the exterior Line, is equal to a Line which falls on the Circle; that Line toucheth the same.

Let be drawn the Secant AC or AH, and let the Rectangle AC, AO, or the Rectangle AH, AF, be equal to the Square of the Line AB; that Line ſhall touch the Circle: draw the touch Line AI (*by the* 17*th.*) and the Line IE.

Demonſtration. Seeing the Line AI toucheth the Circle, the Rectangle AC, AO; or AH, AF, ſhall be equal to the Square of AI. Now the Square of AB, is ſuppoſed equal to each Rectangle: therefore the Squares of AI, and of AB, are equal; and by conſequence the Lines AI, AB. So then the Triangles ABE, AEI, which have all their Sides equal to each other, ſhall be equiangled (*by the* 8*th. of the* 1*ſt.*) and ſeeing the Angle AIE is Right (*by the* 17*th.*) the Line AI being a touch Line, the Angle ABE ſhall be Right, and the Line

Line A B a touch Line alſo (*by the 16th.*)

U S E.

MAurolicus *maketh uſe of this Propoſition to find the Diameter of the Earth. For looking from the top of a Mountain O A, to the extremity of the Earth, along the Line B A; he obſerveth the Angle O A B, which the Line A B, maketh with a plumb Line A C: and he concludes the Length of the Line A B, by a Trigonometrical Calculation. He Multiplieth A B by A B to have its Square, which he divides by A O, the height of the Mountain: from which having taken away A O there remains O C, the Diameter of the Earth, this Propoſition ſerveth alſo to prove the Fifth Propoſition of the Third Book of Trigonometry.* Prop. XXXVII.

The end of the Third Book.

THE

THE FOURTH BOOK OF Euclid's *Elements*.

THis Book is very useful in Trigonometry, seeing that by inscribing of Polygons in a Circle, we have the practice of making the Table of Subtendants, of Sines, of Tangents, and of Secants; which are very necessary in all sorts of measuring.

Secondly, in inscribing of Polygons in a Circle, we have the diversity of the Aspects of the Planets, which take their Names from the same Polygons.

Thirdly, in the practice hereof we have a method which giveth the Quadrature of the Circle, as near the truth as shall be needful. We demonstrate also that a Circle is in duplicate reason to its Diameter.

Fourthly,

Fourthly, Military Architecture hath need of Inscribing Polygons into Circles, in the designing or drawing of regular Fortifications.

The DEFINITIONS.

1. A Right Lined figure is inscribed in a Circle; or the Circle is described about a figure, when all its Angles are in the Circumference of the same Circle. *As the Triangle A B C is inscribed in a Circle, and the Circle is described about the Triangle, because the Angles A, B, C, do touch the Circumference, the Triangle D E F is not inscribed in the Circle, because the Angle D, doth not touch the Circumference of the Circle.* Fig. I. Plate V.

2. A Right Lined Figure is described about a Circle, and the Circle is inscribed within the same figure, when all the sides of the Figure touch the Circumference of the Circle. *As the Triangle G H I, is described about the Circle K L M, because its Sides touch the Circle in K, L, M.* Fig. I.

3. A Line is fitted or inscribed in a Circle, when it is terminated at both ends by the Circumference of the Circle:

Circle: *As the Line N O. The Line R P is not inſcribed in the Circle.*

PROPOSITION I.

PROBLEM.

TO Inſcribe in a Circle a Line, which ſurpaſſeth not its Diameter.

It is propoſed to inſcribe in a Circle A E B D, a Line which ſurpaſſeth not its Diameter, take the length thereof on the Diameter, and let it be for Example B C. Put the Foot of the Compaſs in B, and deſcribe a Circle at the extent of B C, which cutteth the Circle A E B D in D, and E. Draw the Line B D, or B E. It is evident they are equal to B C, *by the Definition of a Circle.*

USE.

THis Propoſition is neceſſary for the practice of thoſe which follow.

PROPOSITION II.

PROBLEM.

TO Inſcribe in a Circle, a Triangle equiangled to another Triangle.

The Circle E G H is propoſed, in which one would inſcribe a Triangle equiangled to the Triangle A B C. Draw the touch Line F E D, (*by the 17th. of the 3d.*) and make at the Point of touching E, the Angle D E H, equal to the Angle B; and the Angle F E G, equal to the Angle C, (*by the 23d. of the 1ſt.*) Draw the Line G H, the Triangle E G H ſhall be equi Angled to A B C.

Demonſtration. The Angle D E H is equal to the Angle E G H, of the Alternate Segment (*by the 32d. of the 3d.*) now the Angle D E H was made equal to the Angle B, and by conſequence the Angles B and G are equal. The Angles C and H, are alſo equal, for the ſame reaſon; and (*by the Coroll. 2. of the 32d. of the 1ſt.*) the Angles A and G E H, ſhall be equal. Therefore the Triangles E G H, A B C, are equiangled.

PRO-

PROPOSITION III.

PROBLEM.

TO describe about a Circle, a Triangle equiangled to another.

If one would describe about a Circle G K H, a Triangle equiangled to A B C; one of the Sides B C, must be continued to D and E; and make the Angle G I H equal to the Angle A B D: and H I K equal to the Angle A C E, then draw the Tangents L G M, L K N, N H M, through the Points G, K, H. The Tangents shall meet each other; for the Angles I K L, I G L, being Right, if one should draw the Line K G, which is not drawn; the Angles K G L, G K L, would be less than two Right: therefore (*by the* 11*th. Axiom*,) the Lines G L, K L, ought to concur.

Demonstration: All the Angles of the Quadrilateral G I H M, are equal to four Right; seeing it may be reduced into Two Triangles; the Angles I G M, I H M, which are made by the Tangents, are Right; thence the Angles M and I are equivalent to Two Right as well as the Angles A B C, A B D. Now the

Angle

Angle GIH, is equal to the Angle ABD by construction: therefore the Angle M shall be equal to the Angle ABC. For the same reason the Angles N and ACB are equal; and so the Triangles LMN, ABC, are equiangled.

PROPOSITION IV.

PROBLEM.

To inscribe a Circle in a Triangle.

IF you would inscribe a Circle in a Triangle ABC; divide into Two equally the Angles ABC, ACB, (*by the 9th. of the 1st.*) drawing the Lines CD, BD, which concurr in the Point D. Then draw from the Point D, the Perpendiculars DE, DF, DG; which shall be equal; so that the Circle described on the point D, at the opening DE, shall pass through F and G.

Demonstration. The Triangles DEB, DBF, have the Angles DEB, DFB, equal, seeing they are Right; the Angles DBE, DBF, are also equal, the Angle ABC having been divided into Two equally; the Side DB is

common:

common: therefore (*by the 26th. of the 1ſt.*) theſe Triangles ſhall be equal in every reſpect; and the Sides D E, D F, ſhall be equal. One might demonſtrate after the ſame manner, that the Sides D F, D G, are equal. One may therefore deſcribe a Circle which ſhall paſs through the Points E, F, G; and ſeeing the Angles E, F, G, are Right, the Sides A B, A C, B C, touch the Circle, which ſhall by conſequence be inſcribed in the Triangle.

PROPOSITION V.

PROBLEM.

To deſcribe a Circle about a Triangle.

IF you would deſcribe a Circle about a Triangle A B C; divide the Sides A B, B C, into Two equally, in D and E, drawing the Perpendiculars D F, E F, which concurr in the Point F. If you deſcribe a Circle on the Center F, at the opening F B, it ſhall paſs through A and C; that is to ſay, that the Lines F A, F B, F C, are equal.

Demonſtration.

Demonſtration. The Triangles ADF, BDF, have the Side DF, common, and the Sides AD, DB, equal, ſeeing the Side AB hath been divided equally; and the Angles in D are equal, being Right. Thence (*by the 4th of the 1ſt.*) the Baſes AF, BF, are equal; and for the ſame reaſon the Baſes BF, CF.

USE.

WE have often need to inſcribe a Triangle in a Circle; as in the firſt Propoſition of the Third Book of Trigonometry. This practice is neceſſary for to meaſure the Area of a Triangle, and upon ſeveral other occaſions.

PROPOSITION VI.

PROBLEM.

To inſcribe a Square in a Circle.

TO inſcribe a Square in the Circle ABCD; draw to the Diameter AB, the Perpendicular DC, which may paſs through the Center E. Draw alſo the Lines AC, CB, BD, AD; and you will have inſcribed in the Circle the Square ACBD.

Demon-

Demonstration. The Triangles AEC, CEB, have their Sides equal, and the Angles AEC, CEB, equal, seeing they are Right; therefore the Base AC, CB, are equal (*by the 4th of the 1st.*) Moreover, seeing the Sides AE, CE, are equal, and the Angle E being Right, they shall each of them be semi-right, (*by the 32d. of the 1st.*) So then the Angle ECB is semi-right. And by consequence the Angle ACB shall be Right. It is the same of all the other Angles: therefore the Figure ACDB is a Square.

PROPOSITION VII.

PROBLEM.

To describe a Square about a Circle.

HAving drawn the Two Diameters AB, CD, which cut each other perpendicularly in the Center E: draw the touch-Lines FG, GH, HI, FI, through the Points A, D, B, C; and you will have described a Square FGHI, about the Cirtle ACBD.

Demonstration. The Angle E and A are Right; thence (*by the 29th. of the 1st.*) the Lines FG, CD, are parallels. I prove after the same manner, that GH,

HI,

HI; FI, AB; AB, GH, are parallels; thence the Figure FCDG, is a parallelogram: and *(by the 34th. of the 1ſt.)* the Lines FG, CD, are equal; as alſo GD, IH; FI, AB; AB, GH: and by conſequence the Sides of the Figure FG, GH; HI, IF; are equal. Moreover, ſeeing the Lines FG, CD, are parallels, and that the Angle FCE is Right; the Angle G ſhall be alſo Right, *(by the 29th. of the 1ſt.)* I demonſtrate after the ſame manner, that the Angles F, H, and I, are Right. Therefore the Figure FGHI is a Square, and its Sides touch the Circle.

PROPOSITION VIII.

PROBLEM.

To inſcribe a Circle in a Square.

IF you will inſcribe a Circle in the Square FGHI; divide the Sides FG, GH, HI, FI, in the middle in A, D, B, C; and draw the Lines AB, CD, which cutteth each other in the Point E. I demonſtrate that the Lines EA, ED, EC, EB, are equal; and that the Angles in A, B, C, D, are Right; and

that ſo you may deſcribe a Circle on the Center E, which ſhall paſs through A, D, B, C, and which toucheth the Sides of the Square.

Demonſtration. Seeing the Lines AB, GH, conjoyns the Lines AG, BH, which are parallel, and equal; they ſhall be alſo parallel and equal; therefore the Figure A G H B is a parallelogram, and the Lines AE, GD; AG, ED, being parallel; and AG, GD, being equal; AE, ED, ſhall be alſo equal. It is the ſame with the others AE, EC, EB. Moreover AG, ED, being parallels; and the Angle G being Right, the Angle D ſhall be alſo a Right Angle. One may then on the Center E, deſcribe the Circle ADBC, which ſhall paſs through the Points A, D, B, C, and which ſhall touch the Sides of the Square.

PROPOSITION IX.

PROBLEM.

To deſcribe a Circle about a Square.

TO deſcribe a Circle about a Square AB, FD; draw the Diagonals AE, BD, which cutteth each other

in the Point E. This Point E ſhall be the Center of the Circle, which will paſs through the Points A, F, B, D. I muſt then demonſtrate that the Lines AE, FE, BE, DE, are equal.

Demonſtration. The Sides AB, FB, are equal, and the Angle B is Right: Therefore the Angles FAB, BFA, are equal (*by the 5th. of the 1ſt.*) and ſemi-right, (*by the 32d. of the 1ſt.*) I Demonſtrate after the ſame manner, that the Angles ABD, ADB; FDB, DBF, are Semi-right. So the Triangle AEB, having the Angles EAB, EBA, ſemi-right, and conſequently equal; it ſhall alſo have (*by the 26th. of the 1ſt*) the Sides AE, EB, equal. One might Demonſtrate after the ſame manner, that the Lines EF, EB; EF, ED, are equal.

USE.

WE Demonſtrate in the Twelfth Book that Polygons deſcribed in Circles, degenerate into Circles; and as thoſe Polygons are always in a duplicate Ratio of their Diameters, that Circles are ſo likewiſe. We have alſo need in Practical Geometry to inſcribe a Square and other

Polygons, within and without a Circle, to reduce the Circle into a Square.

PROPOSITION X.

PROBLEM.

TO make an Iſoſceles Triangle which hath each of the Angles on the Baſe double to the Third.

To make the Iſoſceles Triangle ABD, which may have each of its Angle ABD, ADB, double to the Angle A; divide the Line AB (*by the 11th. of the 2d.*) in ſuch manner that the Square AC be equal to the Rectangle AB, BC. Deſcribe on the Center A, at the opening AB, a Circle BD; in which you ſhall inſcribe BD equal to AC. Draw the Line DC, and deſcribe a Circle about the Triangle ACD, (*by the 5th.*)

Demonſtration. Seeing that the Square of CA, or BD, is equal to the Rectangle comprehended under AB, BC; the Line BD ſhall touch the Circle ACD, in the Point D, (*by the 37th. of the 3d.*) thence the Angle BDC ſhall be equal to the Angle A comprehended in the Alternate Segment CAD, (*by the 32d. of the Third.*) Now the Angle BCD exteriour in reſpect

reſpect of the Triangle A C D, is equal to the Angle A and C D B; therefore the Angle B C D is equal to the Angle B D A. Moreover the Angle A D B, is equal to the Angle A B D, (*by the 5th. of the 1ſt.*) thence the Angles B C D, D B C, are equal; and (*by the 6th. of the 1 ſt.*) the Sides B D, D C, ſhall be equal. And ſeeing B D is equal to A C, the Sides A C, C D, ſhall be equal, and the Angles A and C D A ſhall be ſo alſo. Therefore the Angle A D B is double to the Angle A.

PROPOSITION XI.

PROBLEM.

TO inſcribe a Regular Pentagon in a Circle.

To inſcribe a Regular Pentagon in a Circle; deſcribe (*by the 10th*) an Iſoſceles Triangle A B C, which ſhall have the Angles A B C, A C B, on the Baſe, each double to the Angle A. Inſcribe in the Circle, a Triangle D E F, equiangled to the Triangle A B C; divide into Two equally the Angles D E F, D F E, draw the Lines E G, F H. Laſtly, joyn the Lines D H, D G, G F,

EH: and you ſhall have made a Regular Pentagon, that is to ſay, which hath all its Sides equal, as well as all its Angles.

Demonſtration. The Angles DEG, GEF, DFH, HFE, are the halfs of the Angles DEF, DFE, which are each double to the Angle A; and by conſequence the five Arches, which ſerve to them for Baſes, are equal (*by the 26th. of the 3d.*) and the Lines HD, HE, EF, FG, GD, are equal (*by the 29th. of the 3d.*) Secondly, the Angles DGF, GFE, having each for Baſe thereof, the equal Arks ſhall be likewiſe equal. Thence all the Sides, and Angles of the Pentagons are equal.

PROPOSITION XII.

PROBLEM.

TO deſcribe a Pentagon about a Circle.

Inſcribe a Regular Pentagon ABCDE in the Circle (*by the 11th.*) Drawing the Tangents through the Points A, B, C, D, E, (*by the 17th. of the 3d.*) you will have deſcribed a Regular Pentagon about

about the Circle. Draw the Lines FA, FG, FE, FH, FD.

Demonſtration. The touch Lines GA, GE, are equal (*by the Coroll. of the 36th. of the 3d.*) as alſo EH, HD; the Lines FA, FD, are alſo equal by the Definition of a Circle: thence (*by the 8th. of the 1ſt.*) the Triangles FGA, FGE, are equal in every reſpect; and the Angles AFG, EFG, are equal; as alſo the Angles EFH, DFH. And becauſe that the Angles EFA, EFD, are equal (*by the 27th. of the 3d.*) their halfs EFH, EFG, ſhall be equal: and (*by the 26th. of the 1ſt.*) the Triangles EFH, EFG, ſhall be equal in every reſpect; and the Sides EG, EH, alſo equal. I demonſtrate after the ſame manner, that each of the Sides are divided equally into two parts; and by conſequence, ſeeing the Lines AG, GE, are equal; GH, HI, ſhall be alſo equal. Moreover, the Angles G and H, being double to the Angles FGE, FHI, are alſo equal. We have then deſcribed a Regular Pentagon about a Circle.

PROPOSITION XIII.

PROBLEM.

TO Inſcribe a Circle in a Regular Pentagon.

To inſcribe a Circle in the Regular Pentagon A B C D E: divide the Angles A and B into two equally by the Lines A F, B F, which will meet at the Point F. Then drawing the Line F G perpendicular to A B, deſcribe a Circle on the Center F, at the opening F G. I ſay that it will touch all the other Sides, that is to ſay, having drawn F H perpendicular to B C; F G, and F H, ſhall be equal.

Demonſtration. Seeing the equal Angles A and B have been divided into two equally, their halfs G A F, G B F, ſhall be equal; and ſeeing the Angles in G are Right, the Triangles A F G, B F G, ſhall be equal in every reſpect (*by the 26th. of the 1ſt.*) So the Lines A G, G B, are equal. Moreover, I prove that the Lines B G, B H; as well as F G, F H, are equal: And the Sides A B, B C, of a Regular Pentagon being equal; the Lines B H, H C, ſhall be equal.

equal. And by conſequence, the Angles G and H being Right and equal, the Triangles B F H, H F C, ſhall be equal in every reſpect. And the Angles F B H, F C H, ſhall be equal. And ſeeing the Angles B and C are equal, the Angle F C H ſhall be the half of the Angle C. So going from the one to the other, I demonſtrate that all the perpendiculars F G, F H, *&c.* are equal.

PROPOSITION XIV.

PROBLEM.

TO deſcribe a Circle about a Regular Pentagon.

To deſcribe a Circle about a Regular Pentagon A B C D E: divide two of its Sides A B, B C, in G and H; draw the Perpendiculars G F, H F. The Circle drawn on the Center F, at the opening F A, ſhall paſs through B, C, D, E.

Demonſtration. Suppoſe that the Circle be deſcribed already, it is evident (*by the 1ſt. of the 3d.*) that having divided the Line A B in the middle in G; and having drawn the perpendicular G F; the Center of the Circle is in that perpendicular; it is alſo in F H, and

therefore must be in the Point of their interſection at F.

USE.

THeſe Propoſitions are only uſeful to make the Table of Sines, and for tracing out the ground of Cittadels: for Pentagons are the moſt ordinary. You muſt alſo take notice, that theſe ways of deſcribing a Pentagon about a Circle, may be applyed to the other Polygons. I have given another way to inſcribe a Regular Pentagon in a Circle, in Military Architecture.

PROPOSITION XV.

PROBLEM.

TO inſcribe a Regular Hexagon in a Circle.

To inſcribe a Regular Hexagon in the Circle ABCDEF: draw the Diameter AD, and putting the Foot of the Compaſs in the Point D, deſcribe a Circle at the opening DG; which ſhall interſect the Circle in the Points EC, then draw the Diameters EGB, CGF, and the Lines AB, AF, and the others.

Demonſtration. It is evident that the Triangles

Triangles C D G, D G E, are equilateral; wherefore the Angles C G D, D G E, and their oppoſites B G A, A G F, are each of them the third part of two Right; and that is 60 degrees. Now all the Angles which can be made about one Point is equal to four Right; that is to ſay, 360. So taking away four times 60, that is 240, from 360; there remains 120 degrees; for B G C, and F G E, whence they ſhall each be 60 degrees. So all the Angles at the Center being equal; all the Arks and all the Sides ſhall be equal; and each Angle A, B, C, *&c.* ſhall be compoſed of two Angles of Sixty; that is to ſay, One Hundred and Twenty degrees. They ſhall therefore be equal.

Corol. The Side of a Hexagon is equal to the Semi diameter.

USE.

BEcauſe that the Side of an Hexagon is the Baſe of an Ark of Sixty degrees, and that is equal to the Semi-Diameter; its half is the Sine of Thirty; and it is with this Sine we begin the Tables of Sines. Euclid *treateth of Hexagons in the laſt Book of his Elements.*

PRO-

PROPOSITION XVI.

PROBLEM.

TO inscribe a Regular Pentadecagon in a Circle.

Inscribe in a Circle an equilateral Triangle A B C (*by the* 2*d.*) and a Regular Pentagon (*by the* 11*th.*) in such sort that the Angles meet in the Point A. The Lines B F, B I, I E, shall be the Sides of the Pentadecagon; and by inscribing in the other Arks, Lines equal to B F, B I, you may compleat this Polygon.

Demonstration. Seeing the Line A B is the Side of the Equilateral Triangle; the Ark A E B, shall be the third of the whole Circle, or 5 fifteenths. And the Ark A E being the fifth part, it shall contain $\frac{1}{15}$; thence E B contains two: and if you divide it in the middle in I, each part shall be a fifteenth.

USE.

THis Proposition serveth only to open the way for other Polygons. We have in the compass of Proportion, very easie methods

methods to inſcribe all the ordinary Polygons: but they are grounded on this. For one could not put Polygons on that Inſtrument, if one did not find their ſides by this Propoſition, or ſuch like.

The end of the Fourth Book.

THE

THE FIFTH BOOK OF Euclid's *Elements*.

THis Fifth Book is absolutely necessary to demonstrate the Propositions of the Sixth Book. It containeth a most universal Doctrine, and a way of arguing by Proportion, which is most subtile, solid, and Brief. So that all Treatises which are founded on Proportions, cannot be without this Mathematical Logick. Geometry, Arithmetick, Musick, Astronomy, Statick, and to say in one word all the Treatises of the Sciences are demonstrated by the Propositions of this Book. The greatest part of Measuring is done by Proportions, and in practisal Geometry. And one may demonstrate all the Rules of Arithmetick by the Theorems hereof, wherefore it is not necessary to have recourse

recourse to the Seventh, Eighth, or Ninth Books. The Musick of the Ancients is scarce any thing else but the Doctrine of Proportion applyed to the Senses. It is the same in Staticks, which considers the Proportion of Weights. In fine one may affirm, that if one should take away from Mathematicians the knowledge of the Propositions that this Book giveth us, the remainder would be of little use.

DEFINITIONS.

A Lesser quantity compared with a greater, is called a part. *As if one compare the Line C D of Two Foot, with the Line A B of six, it is called a part. And although in effect C D be not in A B, provided the Line A E equal to C D, is found in A B, there is given thereto the name of part.*

The whole corresponds to its part: and this shall be the greater quantity compared with the lesser; whether it contains the same in effect, or that it doth not contain the same.

Parts or quantities taken in general are divided

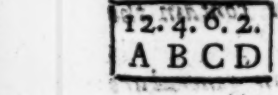
divided ordinarily into Aliquot parts, and Aliquant parts.

1. An Aliquot part (which *Euclid* defines in this Book) is a Magnitude of a Magnitude, the leſſer of the greater, when it meaſureth it exactly. *That is to ſay, that it is a leſſer quantity compared with a greater, which it meaſureth preciſely. As the Line of Two Foot taken Three times, is equal to a Line of Six Foot.*

2. Multiplex is a Magnitude of a Magnitude, the greater of the leſſer, when the leſſer meaſureth the greater: *That is to ſay, that Multiplex is a great quantity compared with a leſſer, which it contains preciſely ſome number of times, For Example, the Line of Six Foot, is treble to a Line of Two Foot.*

Aliquant parts, is a leſſer quantity compared with a greater, which it meaſureth not exactly. So a Line of 4 Foot, is an Aliquant part of a Line of 10 Foot.

Equimultiplexes are Magnitudes which contain equally their Aliquot parts, *that is to ſay the ſame number of times, For example, if A contains as many times B, as C contains D; A and C ſhall be equal Multiplexes of B and D.*



3. Reaſon

3. Reaſon, (or *Ratio*) is a mutual habitude or reſpect of one Magnitude to another of the ſame Species. I have added of the ſame Species.

4. For *Euclid* ſaith, that Magnitudes have the ſame reaſon, when being multiplied, they may ſurpaſs each other. *To do which, they muſt be of the ſame Species. In effect, a Line hath no manner of Reaſon with a Surface, becauſe a Line taken Mathematically is conſidered without any Breadth: ſo that if it be multiplyed as many times as you pleaſe, it giveth no Breadth, and notwithſtanding a Surface hath Breadth.*

Seeing that Reaſon is a mutal habitude or reſpect of a Magnitude to another, it ought to have two terms. That which the Philoſophers would call foundation, is named by the Mathematicians Antecedent, *and the term is called* Conſequent. *As if we compare the Magnitude A, to the Magnitude B, this habitude or Reaſon ſhall have for Antecedent the quantity A, and for conſequent the quantity B. As on the contrary, if we compare the Magnitude B, with A, this Reaſon of B to A, ſhall have for Antecedent the Magnitude B, and for conſequent the Magnitude A.*

The

The Reason or habitude of one Magnitude to another, is divided into rational Reason, and irrational Reason. Rational Reason is a habitude of one Magnitude to another, which is commensurable thereto; that is to say, that those Magnitudes have a common measure, which measureth both exactly. As the reason of a Line of 4 Foot, to a Line of 6, is rational; because a Line of two Foot measureth both exactly: and when this happeneth, those Magnitudes have the same Reason as one Number hath to another. For Example, because that the Line of two Foot, which is the common measure, is found twice in the Line of 4 Foot, and thrice in the Line of 6; the first to the second shall have the same Reason as 2 to 3.

Irrational Reason is between Two Magnitudes of the same Species which are incommensurable, that is to say, that have not a common measure. As the Reason of the Side of a Square to its Diagonal. For there cannot be found any measure, although never so little, which will measure both precisely.

Four Magnitudes shall be in the same Reason, or shall be Proportionals, when the Reason of the first to the second, shall be the same, or like to that of the third to the

fourth

fourth: wherefore to ſpeak properly, Proportion is a ſimilitude of Reaſon. But one findeth it difficult to underſtand in what conſiſteth this ſimilitude of Reaſon. It is only to ſay that two habitudes or Relations be alike. For Euclid *hath not given a juſt Definition, and which might have explained its Nature; having contented himſelf to give us a mark by which we may know, if Magnitude have the ſame Reaſon. And the obſcurity of this Definition hath made this Book difficult. I will endeavour to ſupply this default.*

5. *Euclide* ſaith, that Four Magnitudes have the ſame Reaſon, when having taken the Equi-multiplices of the firſt, and of the third; and other Equi-multiplices of the ſecond, and of the fourth; whatever combination is made, when the Multiplex of the firſt is greater than the Multiplex of the ſecond; the Multiplex of the third ſhall be alſo greater than the Multiplex of the fourth: And when the Multiplex of the firſt is equal, or leſs than the Multiplev of the ſecond; the Multiplex of the third is equal or leſs than the Multiplex of the fourth. That then there is the ſame Reaſon between the firſt and ſecond; as there is between the third and fourth.

As

A	B	C	D
2.	4.	3.	6.
E	F	G	H
10	8.	15	12.
K	L	M	N
8.	8.	12.	12.
O	P	Q	R
6.	16	9.	24

As if there were proposed four Magnitudes A, B, C, D. Having taken the Equi-multiplexes of A and C, which let be E and G, quintuplex: F and H double to B and D. In like manner, taking K and M, quadruple to A and C: L and N double to B and D. Taking again O and Q, triple to A and C: P and R quadruple to B and D. Now because E being greater than F; G is greater than H: and K beng equal to L; M is equal to N: In fine O being lesser than P; Q is lesser than R. Then A shall have the same Reason to B, as C to D. I believe that Euclid *ought to have Demonstrated this Proposition, seeing it is so intangled that it cannot pass for a Maxim.*

To explain well what Proportion is, it is to say, that four Magnitudes have the same Ratio; although one may say in general, that to that end the first must be alike part, or a like whole, in respect of the second; as is the third, compared to the fourth: notwithstanding because this Definition doth not convene with the Reason of equality, there must be given a more general; and to make

make it intelligible, it must be explained what is meant by a like Aliquot part.

Like Aliquot parts are those which are as many times in their whole, as three in respect of nine; two in respect of six, are alike Aliquot parts, because each are found three times in their whole.

The first quantity will have the same Reason to the second, as the third hath to the fourth, if the first contains as many times any Aliquot part of the second whatever, as the 3d. contains alike Aliquot part of the 4th.

A, B, C, D.

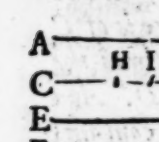
as if A contains as many times a Hundreth, a Thousandth, a Millionth part of B: as C contains a Hundreth, a Thousandth, a Millionth part of D; and so of any other Aliquot parts imaginable: there will be the same Reason of A to B; as of C to D.

To make this Definition yet clearer; I will in the first place prove, that if there be the same reason of A to B, as there is of C to D; A will contain as many times the Aliquot parts of B, as C doth of D. And I will afterwards prove, that if A contains as many times the Aliquot parts of B, as C doth of D; there will be the same Reason of A to B, as of C to D.

The

The first Point seemeth evident enough, provided one doth conceive the terms: for if A contains one Hundred, and one times the tenth part of B; and C only One Hundred times the tenth part of D: the Magnitude A compared with B, would be a greater whole, than C compared with D: so that it could not be compared after the same manner, that is to say, the habitude or Relation would not be the same.

The second point seemeth more difficult; to wit, whether if this propriety be so found, the Reason shall be the same, that is to say, if A B contains as many times any Aliquot parts whatever of C D, as E contains like Aliquot parts of F: there shall be the same Reason of A B to C D, as of E to F. For I will prove that if there were not the same Reason, A would contain more times any Aliquot part of B, than C containeth alike Aliquot part of D: which would be contrary to what we had supposed.



Let us suppose then that there is a greater reason of A B to C D, than of E to F; that is to say, that A B is greater than it ought to be, to have the same Reason to C D, as E hath to F. Where-

fore

fore a leſſer Magnitude than A B, for Example, A G ſhall have the ſame Reaſon to C D, as E to F. Divide C D in the middle in H and H D, in the middle in I and I D, in the middle in K. In continuing this Diviſion you would find an Aliquot part of C D leſs than G B; let it be K D.

Demonſtration. Seeing there is the ſame Reaſon of A G to C D, as of E to F; A G will contain as many times K D an Aliquot part of C D, as E would contain a like Aliquot part of F. Now A B contains K D, once more than A G: thence A B will contain once more K D an Aliquot part of C D, than E doth contain a like Aliquot part of F; which would be contrary to the ſuppoſition.

6. There will be a greater Reaſon of the firſt quantity to the ſecond, than of the third to the fourth: if the firſt contains more times any Aliquot part of the ſecond, than the third doth contain a like Aliquot part of the fourth. *As* 101 *hath a greater Reaſon to* 10 *than* 200 *to* 20; *becauſe that* 101 *contains One Hundred and one times the Tenth part of* 10, *and* 200 *contains only One Hundred times the Tenth part of* 20, *which is* 2.

7. The

7. The Magnitudes or quantities which are in the ſame Reaſon, are called Proportionals.

8. A Proportion or Analogie, is a Similitude of Reaſon or habitude.

9. A Proportion ought to have at leaſt three terms. *For to the end there be ſimilitude of Reaſon, there muſt be two Reaſons: Now each Reaſon having two terms, the antecedent and the conſequent, it ſeemeth there ought to be four, as when we ſay that there is the ſame Reaſon of A to B, as of C to D: but becauſe the conſequent of the firſt Reaſon may be taken for antecedent in the ſecond, three terms may ſuffice, as when I ſay that there is the ſame Reaſon of A to B, as of B to C.*

10. Magnitudes are in continued Proportion, when the Terms between them are taken twice; that is to ſay, as antecedent and as conſequent. *As if there be the ſame Reaſon of A to B, as of B to C, and of C to D.*

11. Then A to C ſhall be in duplicate Ratio of A to B: and the Ratio of A to D ſhall be in triplicate Ratio to that of A to B.

It

It is to be taken notice, that there is a great deal of difference between double Ratio and duplicate Ratio. We say that the Ratio of four to two is double, that is to say, four is the double of two, whence it followeth that the number two, is that which giveth the Name to this Ratio, or rather to the Antecedent of this Ratio. So we we say, double, triple, quadruple, quintuple, which are Denominations taken from those numbers, duo, tres, quatuor, quinque, *compared with unity: for we better conceive a Reason when its terms are small. But as I have already taken notice, those Denominations fall rather on the Antecedent than on the Reason it self: we call that double, triple, Reason or Ratio, when the Antecedent is double or triple to the consequent: but when we say the Reason is duplicate, we mean a Reason compounded of two like Reasons, as if there be the same Reason of two to four, as of four to eight; the Reason of two and eight being compounded of the Reason of two and four, and of that of four and eight, which are alike; and as equal the Reason or Ratio of two to eight, shall be duplicated by each. Three to twenty seven, is a duplicated Reason of that of three to nine. The Reason of two to four is called subduple;*

that is to say, two is the half of four: but the reason of two to eight is duple of the sub-duple; that is to say, that two is the half of the half of eight: as three is the third of the third of twenty seven; where you see there is taken twice the Denominator $\frac{1}{2}$ *and* $\frac{1}{3}$. *In like manner, eight to two is a duplicate reason of eight to four, because eight is double to four, but eight is the double of the double of two. If there be four terms in the same continued Reason, that of the first and last is triple to that of the first and second, as if one put these four Numbers, two, four, eight, sixteen; the reason of two to sixteen is triple of two to four; for two is the half of the half of the half of sixteen. As the reason of sixteen to two, is triple of sixteen to eight; for sixteen is the double of eight, and it is the double of the double of the double of two.*

12. Magnitudes are homologous, the Antecedents to the Antecedents, and the Consequents to the Consequents. *As if there be the same Reason of A to B, as of C to D, A and C are homologous, or Magnitudes of a like Ratio.*

The following Definitions are ways of arguing by Proportion, and it is principally to demonstrate the same that this Book is composed.

13. Al-

13. Alternate Reaſon, or by Permutation, or Exchange, is, when we compare the Antecedents one with the other, as alſo the conſequents. *For example, if becauſe there is the ſame reaſon of A to B as of C to D, I conclude there is the ſame reaſon of A to C as of B to D; this way of reaſoning cannot take place but when the four terms are of the ſame Specie; that is to ſay, either all four Lines, or Superficies, or Solids. Propoſition* 16.

14. Converſe, or Inverſe Reaſon, is a compariſon of the Conſequents to the Antecedents. *As, if becauſe there is the ſame reaſon of A to B as of C to D; I conclude, there is the ſame reaſon of B to A as there is of D to C. Propoſition* 16.

15. Compoſition of Reaſon is a compariſon of the Antecedent and Conſequent taken together, to the Conſequent alone. *As, if there be the ſame Reaſon of A to B as of C to D; I conclude alſo, that there is the ſame reaſon of A B to B as of C D to D. Prop.* 18.

16. Diviſion of Reaſon is a compariſon of the exceſs of the Antecedent above the Conſequent, to the ſame Conſequent. *As, if there be the ſame reaſon of A B to B, as of C D to D; I conclude*

that there is the ſame reaſon of A to B, as of C D. Prop. 17.

17. Converſion of Reaſon is the compariſon of the Antecedent to the difference of the Terms. *As, if there be the ſame reaſon of A B to B, as of C D to D, I conclude, that there is the ſame reaſon of A B to A, as of C D to C. Propoſition* 18.

18. Proportion of Equality is a compariſon of the extream Quantities in leaving out thoſe in the middle. *As if there were the ſame reaſon of A to B as of E to F, and of B to C as of F to G, and of C to D as of G to H. I draw this Conſequence, that there is therefore the ſame reaſon of A to D as of E to H.*

A, B, C, D,
E, F, G, H.

19. Proportion of Equality well ranked, is that in which one compareth the Terms, in the ſame manner of Order; *as in the preceding Example. Prop.* 22.

20. Proportion of Equality ill ranked, is that in which one compareth the Terms with a different Order. *As if there were the ſame reaſon of A to B as of G to H, and of B to C as of F to G, and of C to D as of E to F. I draw this Concluſion, that there is the ſame reaſon of A to D as of E to H. Prop.* 28.

Here

Here is all the ways of arguing by Proportion. There is the same reason of A to B as of C to D; therefore by alternate reason there is the same reason of A to C as of B to D: and by inverſed reason, there is the same Reason of B to A as of D to C; and by compoſition, there is the same reason of A B to B as of C D to D.

By Diviſion of Reason, if there be the same reason of A B to B as of C D to D, there is the same reason of A to B as of C to D; and by Converſion, there is the same reason of A B to A as of C D to C.

By reason of Equality well ranked, if there be the same reason of A to B as of C to D, and alſo the same reason of B to E as of D to F; there will be the same reaſon of A to E as of C to F.

By reason of Equality ill ranked, if there be the same reason of A to B as of D to F, and alſo the same reason of B to E as of C to D; there will be the same reason of A to E as of C to F.

This Book contains twenty five Propoſitions of Euclid, *to which there has been added ten, which are received. The firſt ſix of this Book are uſeful only to prove the following Propoſitions by the method of Equimultiplices: and ſeeing I do not make uſe of that method, I begin at the ſeventh,*

without changing the Order or Number of the Propoſitions.

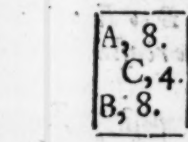
The Demands or Propoſitions.

Three Magnitudes A B C being propoſed, it is required, that it be agreed to, or granted, that there is a fourth Magnitude in Poſſibility, to which the Magnitude C hath the ſame Reaſon as the Magnitude A hath to the Magnitude B.

PROPOSITION VII.

THEOREM.

THe Magnitudes which are equal have the ſame reaſon to a third Magnitude; and one and the ſame Magnitude hath the ſame reaſon to equal Magnitudes.



If the Magnitude A and B are equal, they ſhall have the ſame reaſon to a third Magnitude C.

Demonſtration. If one of the two, for example A, ſhould have a greater Reaſon to the magnitude C, than B; A would contain more times a certain aliquot part of C, than B would contain: thence A would be greater than B, contrary to what we have ſuppoſed.

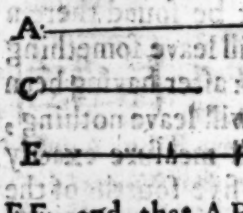
Secondly, I ſay if A and B are equal, the Magnitude C ſhall have the ſame Reaſon to the Magnitude A, as it hath to the Magtitude B.

Demonſtration. If the Magnitude C ſhould have a greater Reaſon to the Magnitude A, than it hath to the Magnitude B; it ought to contain more times an Aliquot of the Magnitude A, than it doth contain a like Aliquot part of the Magnitude B. So that, that part of A ought to be leſs than a like Aliquot part of B. Thence A ſhould be leſs than B: which is contrary to the ſuppoſition.

PROPOSITION VIII.

THEOREM.

THe greateſt of Two Magnitudes, hath a greater Reaſon to the ſame third, than the leſſer; and the ſame Third hath a leſſer Reaſon to the greater Magnitude than it hath to the leſſer.



I ſuppoſe that one ſhould compare the Magnitudes A B and C, with the ſame third EF; and that AB be greater than C. I ſay that there is a greater Reaſon of

A B to E F, than of C to E F. Let A D be taken equal to C: and let E F be divided in the middle, and the half again in the middle, and ſo continuedly untill ſuch time one find G F, an Aliquot part of E F, to be leſs than D B.

Demonſtration. A D, and C are equal; thence there is the ſame Reaſon of A D to E F, as there is of C to E F (*by the 7th.*) and (*by the 4th. Definition*) A D ſhall contain as many times G F an Aliquot part of E F, as C containeth the ſame. Now A B containeth the ſame once more, ſeeing D B is greater than G F: whence (*by the 5th. Definition.*) the Reaſon of A B to E F, is greater than that of C to the ſame third E F.

Secondly I ſay, that E F hath a leſſer Reaſon to A B, than to the Magnitude C. Let there be taken any Aliquot part of C, for Example the one Fourth, as many times as it can be taken in E F. Let us ſuppoſe that it be found therein five times, either it will leave ſomething of the Magnitude E F: after having been taken five times. Or will leave nothing, that is to ſay, it will meaſure exactly E F, it is evident that five fourths of the Magnitude A B will be a greater Line, than the Fourth of C taken five times:

ſo

so that it cannot be found to be five times in E F. And if the fourth of C taken five times, falleth short as in G: either the fourth of A B taken five times, will reach to F; or fall short in I. If it reacheth to F; there will be the same Reason of E F to A B, as of E G to C: by the preceding Argument, E F to C hath a greater Reason to C, than to A B. That if the Quarter of A B taken five times reacheth to I; there will the same Reason of E I to A B, as of E G and C. Now E I, or E F hath a greater Reason than E G to C. Therefore E F to C hath a greater Reason than the same E F to A B.

PROPOSITION IX.

THEOREM.

MAgnitudes are equal when they have the same Reason to a Third Magnitude.

A, B, C, 12.12.16.

If the Magnitudes A and B have the same Reason to a Third Magnitude C: I say A and B are equal.

Demonstration. If one of the Two: for Example A, was greater than B; it would have a greater Reason to the Mag-

nitude C (*by the 8th.*) which would be contrary to the Hypotheſis.

Secondly, if the Magnitude C hath the ſame Reaſon to the Magnitude A, as it hath to the Magnitude B: I ſay that A and B are equal, for if A was greater than B; C would have a greater Reaſon to the Magnitude B, than to the Magnitude A; which would be alſo contrary to the ſuppoſition.

PROPOSITION X.

THEOREM.

THe Magnitude which hath the greateſt Reaſon to the ſame Magnitude, is the greateſt, and that is the leaſt to which the ſame hath the greateſt Reaſon.

A, B, C. If there be a greater Reaſon between A and C, than between B and C: I ſay that A is greater than B, for if A and B were equal, they would have the ſame Reaſon to C: if A were leſs than B, there would be a greater Reaſon between B and C, than between A and C. The one and the other is contrary to the Hypotheſis.

Secondly, if there be a leſſer Reaſon between C and A, than between C and

B. I ſay that A ſhall be greater than B. For if A and B were equal, C would have the ſame Reaſon to both (*by the 8th.*) If A was leſs than B, C would have leſs Reaſon to B than to A, the one and the other is contrary to what we have ſuppoſed.

PROPOSITION XI.

THEOREM.

THe Reaſons which are equal to the ſame Third, are alſo the ſame one to another.

A, B, C, D, E, F.
4. 2. 8. 4 6. 3.

If there be the ſame Reaſon between A and B, as between C and D, and if there be alſo the ſame Reaſon between C and D, as between E and F: I ſay there will be the ſame Reaſon betwen A and B; as between E and F.

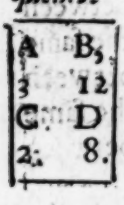
Demonſtration. Seeing there is the ſame Reaſon between A and B, as there is between C and D; A ſhall contain as many times any Aliquot part whatever of B as C containeth alike Aliquot part of D, (*by the 5th. Definition.*) and in like manner, as many times as C containeth

taineth that Aliquot part of D; E contains alike Aliquot part of F. So then, as many times as A containeth any Aliquot part whatever of B; E shall contain also a like Aliquot part of F. Therefore there will be the same Reason between A and B; as there is between E and F.

PROPOSITION XII.

THEOREM.

IF several Magnitudes are Proportionals, there shall be the same Reason of one Antecedent to its consequent, as of all the Antecedents taken together, to their consequents.



If there be the same Reason of A to B, as there is of C to D: I say that there is the same Reason of A and C, taken together to B and D, as of A to B.

Demonstration. Seeing there is the same Reason of A to B, as of C to D; the Magnitude A shall contain as many times any Aliquot part whatever of B, as C contains a like Aliquot part of D; for Example, the fourth (*by the 5th. Definition.*)

nition.) Now the fourth of B, and the fourth of D, are one fourth of B D: ſo then A C ſhall contain as many times one fourth of B D, as A contains the fourth of B: and what I ſay of the fourth verifieth it ſelf of all other aliquot parts. There is therefore the ſame reaſon of A to B, as of A G to B D.

PROPOSITION XIII.

THEOREM.

IF of two equal Reaſons the one is greater than a third, the other ſhall be ſo likewiſe.

AB : CD : EF. If there be the ſame reaſon of A to B, as of C to D, and that there is a greater reaſon of A to B, than of E to F; I ſay that there ſhall be a greater reaſon of C to D, than of E to F.

Demonſtration. Seeing there is a greater reaſon of A to B, than of E to F, A ſhall contain more times any aliquot part of B, than E contains a like aliquot part of F, *(by the 6th. Definition.)* Now C contains a like aliquot part of D, as many times as A contains that of B, ſeeing there is the ſame reaſon of A to B, as of C to

C to D: ſo then C contains an aliquot part of D, more times than E containeth a like aliquot part of F; thence there is a greater reaſon of C to D, than of E to F.

PROPOSITION XIV.

THEOREM.

IF there be the ſame Reaſon of the firſt Magnitude to the ſecond, as of the Third to the fourth: if the firſt be greater, equal, or leſs than the third, the ſecond ſhall be greater, equal, or leſs than the fourth.

|A, B, C, D| If there be the ſame reaſon of A to B, as of C to D; I ſay in the firſt place, that if A be greater than C, B ſhall alſo be greater than D.

Demonſtration. Seeing that A is greater than C, there will be *(by the 9th.)* a greater Reaſon of A to B, than of C to B. Now as A is to B, ſo is C to D; therefore there ſhall be a greater reaſon of C to D, than of C to B; and by conſequence *(according to the tenth)* B ſhall be greater than D.

I ſay in the ſecond place, that if A be equal to C, B ſhall be alſo equal to D.

De-

Demonſtration. Seeing that A and C are equal, there will be the ſame reaſon of A to B, as of C to B, (*by the 7th.*) Now as A is to B, ſo is C to D; thence there is the ſame reaſon of C to B, as of C to D; and by conſequence, B and D are equal (*by the ninth.*) and in the third place if A be leſs than C, B ſhall be alſo leſs than D.

Demonſtration. Seeing that A is leſs than C, there will be a leſſer reaſon of A to B, than of C to B, (*by the 8th.*) Now as A is to B, ſo is C to D; thence there will be a leſſer reaſon of C to D, than of C to B; and by conſequence (*according to the 10th.*) B ſhall be leſs than D.

PROPOSITION XV.

THEOREM.

THE Equimultiplices and like aliquot parts, are in the ſame reaſon.

A,	B,	C,	D.
2.	3.	6.	9.
	E,	2. H,	3.
	F,	2. I,	3.
	G,	2. K,	3.

If the Magnitudes C and D are Equimultiplices of A and B, their aliquot parts, there ſhall be the ſame reaſon of A to B, as of

of C to D: let the Magnitude G be divided into parts equal to A, which shall be E, E, G: let the Magnitude D be divided into parts equal to B, seeing that C and D are Equimultiplices of A and B: there will be as many parts in the one as in the other.

Demonstration. There is the same reason of E to H, of F to I, of G to K, as of A to B; seeing they are equal, thence (*by the* 12*th.*) there shall be the same reason of E, F, G, to H I K; that is to say, of C to D, as of A to B.

Coroll. The same number of aliquot parts of two Magnitudes, are in the same reason with those Magnitudes: for seeing there is the same reason of E to H, as of C to D, and F to I; there will be the same reason of E F to H I, as of C to D.

PROPOSITION XVI.

THEOREM.

Alternate Reason.

IF four Magnitudes of the same Species are proportional, they shall also be proportional alternately.

If

A, B, C, D. 12. 8. 9. 6.

If there be the same reason of A to B, as of C to D, and if the four Magnitudes be of the same Species; that is to say, if all four be Lines, or all four Superficies, or all four Solids, there will be the same reason of A to C, as of B to D. Now suppose that there is a greater reason of A to C than of B to D.

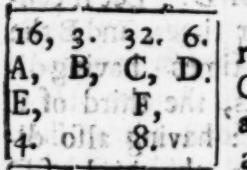
Demonstration. Seeing one would have it, that there is a greater reason of A to C, than of B to D, the Magnitude A will contain an aliquot part of C, for Example the third, more times than B contains the third of D. Let A contain the third of C four times, and B the third of D only three times; having divided A into four parts, the third of C shall be once in each: having also divided B into four parts, the third of D shall not be in each. Thence the three Fourths of A shall contain the three Thirds of C, that is to say, the Magnitude C; and the three Fourths of B will not contain the three Thirds of D, that is to say, the Magnitude D. Again, seeing there is the same reason of A to B, as of C to D, there will be also the same reason of the three Fourths of A to the three Fourths of B, as of C to D, (*by the*

Co-

Coroll. of the 15th.) and (*by the 14th.*) if the three Fourths of A be greater than C, the three fourths of B ſhall be greater than D; although we have demonſtrated the contrary.

LEMMA.

IF there be the ſame reaſon of the firſt Magnitude to the ſecond, as of the third to the fourth; an aliquot part of the firſt ſhall have the ſame reaſon to the ſecond as a like aliquot part of the third hath to the fourth.



If there be the ſame reaſon of A to B as of C to D, and that E be an aliquot part of A, and F a like aliquot part of C; I ſay there is the ſame reaſon of E to B, as of F to D.

Demonſtration. If there were a greater reaſon of E to B than of F to D, E would contain an aliquot part of B more times than F contains a like aliquot part of D. Then E, taken twice, thrice, or four times, would contain an aliquot part of B, more times than F taken twice, thrice, or four times, would contain

contain an aliquot part of D. Now E, taken four times, is equal to A, as F is to C. So A would contain an aliquot part of B more times than C containeth a like aliquot part of D: thence there would be a greater reason of A to B than of C to D; which is contrary to the Supposition.

COROLLARY.

Which in Euclid *is after the fourth Proposition.*

Converse Reason.

IF there be the same Reason of the first to the second, as of the third to the fourth; there shall also be the same Reason of the second to the first, as of the fourth to the third.

A,	B,	C,	D,
4.	8.	12.	24.
E,		F,	
1.		3.	

If there be the same reason of the Magnitude A to B, as of C to D; there will be the same reason of B to A as of D to C.

Demonstration. If there were a greater reason of B to A than of D to C, B would contain an aliquot part of A, for

for example the fourth E, more times than D, contains F the fourth of C. Let us ſuppoſe that B ſhould contain eight times the magnitude E, and D ſhould only contain ſeven times the magnitude F, and ſeeing there is the ſame reaſon of A to B, as of C to D, there will be alſo the ſame reaſon of E to B, as of F to D. (*by the foregoing Lemma*) and (*by the* 15*th.*) E taken eight times ſhall have the ſame Reaſon to B, as F taken eight times to D. Now E taken eight times is contained in B: thence F taken eight times ſhall be contained in D; although we have demonſtrated the contrary. There is not then a greater reaſon of B to A, than of D to C.

PROPOSITION XVII.

THEOREM.

Diviſion of Reaſon.

IF Magnitudes compounded be proportional, they ſhall be proportional alſo when divided.

A, B, C, D.
5. 3. 10. 6.

If there be the ſame reaſon of A B to B, as of C D to D, there ſhall

shall be also the same reason of A to B, as of C to D.

Demonstration. Seeing it is supposed that there is the same reason of A B to B, as of C D to D, A B will contain an aliquot part of B, as many times as C D contains a like aliquot part of D: now this aliquot part is found as many times in B, as the like is found in D: thence taking away B from A B, and D from C D, A will yet have as many aliquot parts of B as C containeth like aliquot parts of D; and by consequence there shall be the same reason of A to B as of C to D.

PROPOSITION XVIII.

THEOREM.

Composition of Reason.

IF Magnitudes divided be proportional, the same also being componnded shall be proportional.

A, B, C, D.
5. 3. 10. 6.

If there be the same reason of A to B as of C to D, there shall be also the same reason of A B to B as of C D to D.

De-

Demonſtration. Seeing it is ſuppoſed that there is the ſame reaſon of A to B as of C to D, A ſhall contain an aliquot part whatever of B, as many times as C contains a like aliquot part of D. Now the Magnitude B contains any aliquot part of it ſelf as many times as D contains alike of it ſelf: thence adding B to A, and D to C, A B ſhall contain an aliquot part of B, as many times as C D contains a like aliquot part of D: there is therefore (*by the fifth Definition*) the ſame reaſon of A B to B, as of C D to D.

COROLLARY.

Converſion of Reaſon.

IF there be the ſame reaſon of A B to B, as of C D to D, there ſhall alſo be the ſame reaſon of A B to A, as of C D to C: for (*by the foregoing*) there will be the ſame reaſon of A to B as of C to D: and (*by the Coroll. of the 16th.*) there will be the ſame reaſon of B to A, as of D to C: and by Compoſition there will be the ſame reaſon of A B to A, as of C D to C.

USE

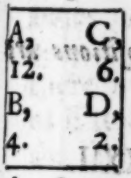
USE.

WE make use of this way of Arguing in almost all the Parts of the Mathematicks.

PROPOSITION XIX.

THEOREM.

IF the Wholes are in the same reason as the Parts taken away, those which remain shall be in the same reason.



If there be the same reason of the Magnitude A B to the Magnitude C D, as of the part B to the part D; I demonstrate, that there shall be the same reason of A to C as of A B to C D.

Demonstration. It is supposed that there is the same reason of A B to C D, as of B to D: thence, by Exchange (*according to the 16th.*) there shall be the same reason of A B to B, as of C D to D; and by Conversion of reason, there shall be the same reason of A B to A, as of C D to C: and again, by Exchange there shall be the same reason of A B to C D, as of A to C.

USE

USE.

IN the Rule of Fellowship this Proposition is sometimes made use of; for they do not make use of the Rule of Three for every Associate, but do content themselves for to give to the last the Remainder of the Gain; supposing, that if there be the same Reason of the whole Sum of the Stock to the whole profit, as of the Stock of one Associate, to his part of the Gain; there will be the same reason of the Stock remaining, to the Remainder of the Gain.

The 20th. and 21th. Propositions are not necessary.

PROPOSITION XXII.

THEOREM.

The reason of Equality with Order.

IF one should propose certain terms to which one should compare a like number in such sort, that those which correspond in the same Ranks be proportional, the first and last shall be proportional.

If

12. 6. 2. 6. 3. 1. A, B, C, D, E, F. If the Magnitudes A, B, C, and the Magnitudes D, E, F, are proportional; that is to say, that there be the same Reason of A to B, as of D to E; of B to C, as of E to F: there will be also the same Reason of A to C, as of D to F.

Demonstration. If there were a greater Reason of A to C, than of D to F; A would contain an Aliquot part of C, for example the half, more times than D would contain the half of F: Let us suppose that the half of C is Twelve times in A, and that the half of F is only Eleven times in D. Now because there is the same Reason of B to C, as of E to F; the Magnitude B will contain the half of C, as many times as E contains the half of F: Let us suppose that those halfs are found Six times in B and E. A which contains Twelve times, the half of C, will have a greater Reason to B, which contains six times the half of C than D, which contains only Eleven times the half of, F, to E, which containeth the same six times; there shall be therefore a greater Reason of A to B, than of D to E, although we had supposed the contrary.

PROPOSITION XXIII.

THEOREM.

THE Reaſon of equality without order, if Two Ranks of terms are in the ſame Reaſon ill Ranked the firſt and the laſt of the one and of the other ſhall be proportional.

A, B, C.	D, E, F, G.
12, 6, 3.	8 4, 2, 1.

If the Magnitudes A, B, C, and the others D, E, F, in like number be in the ſame Reaſon ill Ranked, that is to ſay, that there be the ſame Reaſon of A to B, as of E to F; and the ſame Reaſon of B to C, as of D to E: there will be the ſame Reaſon of A to C, as of D to F. Let there be the ſame Reaſon of B to C, as of F to G.

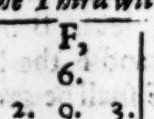
Demonſtration. Seeing there is the ſame Reaſon of A to B, as of E to F; and of B to C, as of F to G: there ſhall be alſo the ſame Reaſon of A to C, as of E to G (*by the* 22*d*) Moreover, ſeeing there is the ſame Reaſon of B to C as of D to E, and of F to G; there ſhall be (*by the* 11*th.*) the ſame Reaſon of D to F, as of E to G: Now as E is to G,

G, ſo is A to C, as we have already proved; thence as A is to C, ſo is D to F.

PROPOSITION XXIV.

THEOREM.

IF there be the ſame Reaſon of the firſt Magnitude to the Second, as of the Third to the Fourth; and the ſame of the Fifth to the Second, as of the Sixth to the Fourth: there will be the ſame Reaſon of the Firſt with the Fifth to the Second, as of the Third with the Sixth to the Fourth.



If there be the ſame Reaſon of A to B as of C to D, of E to B as of F to D; there ſhall be the ſame Reaſon of A E to B, as of C F to D.

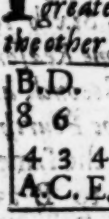
Demonſtration. Seeing there is the ſame Reaſon of A to B as of C to D, A will contain any Aliquot part of B whatever, as many times as C contains a like Aliquot part of D (*by the 5th. Def.*) In like manner E will contain the ſame Aliquot part of B, as many times as F will contain a like Aliquout part of D: So then A E will contain any Aliquot part of B whatever, as many times as C and F will contain

a like Aliquot part of D. There ſhall thence be the ſame Reaſon of AE to B, as of CF to D.

PROPOSITION XXV.

THEOREM.

IF Four Magnitudes are proportional, the greateſt and the leaſt will be greater than the other Two.



If the four Magnitudes AB, CD, E, F are proportional; let AB be the greateſt, and F the leaſt: AB and F ſhall be greater than CD and E.

Seeing there is the ſame Reaſon of AB to CD, as of E to F; and ſeeing it is ſuppoſed that AB is greater than E; CD ſhall be greater than F (*by the* 14*th.*) take from AB the Magnitude A equal to E; and from CD the Magnitude C equal to F.

Demonſtration. Seeing there is the ſame Reaſon of AB to CD, as of A to C; there will be alſo the ſame Reaſon of B to D, as of AB to CD, (*by the* 19*th.*) and AB being ſuppoſed greater than CD; B will be greater than D.

D. Now if A and E be added which are equal, E and F which are alſo equal: A and F ſhall be equal to C and E. And adding to the firſt, B which is greateſt; and to the ſecond D which is leſſer; A B and F ſhall be greater than C D and E.

USE.

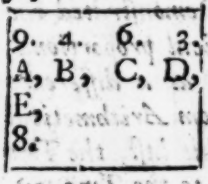
IN this Propoſition is Demonſtrated one Propriety of Geometrical proportionality which ſerveth thereto as a difference which diſtinguiſheth it from Arithmetical proportionality; for in this laſt, the Two middle Terms are equal to the Two extreams; and in the Geometrical the greater and leſſer are greater.

Then the other Two, although the Nine following Propoſitions are not of Euclid, *I thought I ought not to omit them, becauſe ſeveral do make uſe of them, and do Cite them as if they were thereof.*

PROPOSITION XXVI.

THEOREM.

IF there be a greater Reason of the first to the second, than of the Third to the fourth; the fourth will have a greater Reason to the third, than the second to the first.



If there be a greater Reason of A to B, than of C to D; there shall be a greater Reason of D to C, than of B to A. Let us suppose there were the same Reason of E to B, as of C to D: A shall be greater than E, (*by the* 10*th*.)

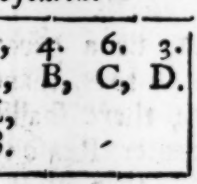
Demonstration. There is the same Reason of E to B, as of C to D; thence (*by the Coroll. of the* 10*th*.) there shall be the same Reason of D to C, as of B to E. Now B hath a greater Reason to E, than to A, (*by the* 8*th*.) thence there is a greater Reason of D to C, than of B to A.

PRO.

PROPOSITION XXVII.

THEOREM.

IF there be a greater Reaſon of the firſt to the ſecond, than of the third to the fourth, there ſhall be alſo a greater Reaſon of the firſt to the third, than of the ſecond to the fourth.



If there be a greater Reaſon of A to B, than of C to D; I demonſtrate that there ſhall be a greater Reaſon of A to C, than of B to D. Let there be the ſame Reaſon of E to B, as of C to D; A ſhall be greater than E.

Demonſtration. There is the ſame Reaſon of E to B, as of C to D; thence (*by the* 16*th.*) there ſhall be the ſame Reaſon of E to C, as of B to D. And becauſe A is greater than E, the Reaſon of A to C ſhall be greater than that of E to C. There is therefore a greater Reaſon of A to C, than of B to D.

PROPOSITION XXVIII.

THEOREM.

IF there be a greater Reaſon of the firſt to the ſecond, than of the third to the fourth; there ſhall be alſo a greater Reaſon of the firſt and ſecond to the ſecond, than of the third and fourth to the fourth.

9. 4. 6. 3.
A, B, C, D,
E,
8.

If there be a greater Reaſon of A to B, than of C to D; there ſhall be alſo a greater Reaſon of AB to B, than of CD to D. Let us ſuppoſe there is the ſame Reaſon of E to B, as of C to D.

Demonſtration. There is the ſame Reaſon of E to B, as of C to D: Thence (*by the* 18*th*.) there ſhall be the ſame Reaſon of EB to B, as of CD to D. and AB being greater than EB, there ſhall be a greater Reaſon of AB to B, than of EB to B; and by conſequence than of CD to D.

PROPOSITION XXIX.

THEOREM.

IF the first and second have a greater Reason to the second than the third and fourth to the fourth; the first to the second shall have a greater Reason than the third to the fourth.

9. 4. 6. 3. A, B, C, D, E, 8.

If there be a greater Reason of A B to B, than of C D to D; there shall be also greater Reason of A to B, than of C to D. Let us suppose that the Reason of E B to B is the same with that of C D to D; E B shall be less than A B, and E less than A.

Demonstration. It is supposed that E B is to B in the same Reason as C D is to D: thence in dividing (*by the 17th.*) there shall be the same Reason of E to B, as of C to D: And A being greater than B, the Reason of A to B shall be greater than that of C to D.

PROPOSITION XXX.

THEOREM.

IF the first and second have a greater Reason to the second, than the third and fourth to the fourth: The first and second shall have a lesser Reason to the first, than the third and fourth to the third.

9.	4.	6.	3.
A,	B,	C,	D.

If A B have a greater Reason to B than C D to D: A B shall have a lesser Reason to A, than C D to C.

Demonstration. We suppose that the Reason of A B to B, is greater than that of C D to D; there shall be a greater Reason of A to B, than of C to D, (*by the* 29*th.*) and (*by the* 26*th.*) there shall be a greater Reason of D to C, than of B to A. Thence by composition (*by the* 28*th.*) the Reason of C D to C shall be greater than of A B to A.

PRO-

PROPOSITION XXXI.

THEOREM.

IF ſeveral Magnitudes are in a greater Reaſon than a like number of other Magnitudes Ranked in the ſame order, the firſt of the firſt Rank ſhall have a greater Reaſon to the laſt, than the firſt of the ſecond Rank to the laſt.

16. 10. 3.	9. 6. 2.
A, B, C,	D, E, F.

If there be a greater Reaſon of A to B, than of D to E; and if B have a greater Reaſon to C, than E to F; there ſhall be a greater Reaſon of A to C, than of D to F.

Demonſtration. Seeing there is a greater Reaſon of A to B than of D to E: there ſhall be alſo a greater Reaſon of A to D, than of B to E. And becauſe there is a greater Reaſon of B to C, than of E to F; there ſhall be alſo a greater Reaſon of B to E, than of C to F. Thence there ſhall be a greater Reaſon of A to D, than of C to F; and by exchange (*by the 27th.*) there will be a greater Reaſon of A to C, than of D to F.

PRO-

PROPOSITION XXXII.

THEOREM.

IF ſeveral Magnitudes are in a greater Reaſon than a like number of other Magnitudes Ranked after another manner: The firſt of the firſt Rank ſhall have a greater Reaſon to the laſt, than the firſt of the ſecond Rank to the laſt.

B,	F,		
12.	3.		
13. 6.	2.	4. 2. 1.	
A,	C, E,	H, I, K,	

If there be a greater Reaſon of A to C, than of I to K: and if C have a greater reaſon to E, than H to I; the Reaſon of A to E, ſhall be greater than the Reaſon of H to K. Let us ſuppoſe that B hath the ſame Reaſon to C, as I to K; A ſhall be greater than B: In like manner, let there be the ſame Reaſon of C to F, as of H to I; F ſhall be greater than E.

Demonſtration. Seeing we ſuppoſe that there is the ſame Reaſon of B to C, as of I to K; and of C to F, as of H to I: there will be alſo the ſame Reaſon of B to F, as of H to K (*by the* 23*d.*) Now there is a greater Reaſon of A to F than of B to F (*by the* 8*th.*) and the Reaſon

Reason of A to E is greater than that of A to F; seeing F is greater than E, there shall thence be a greater Reason of A to E, than of H to K.

PROPOSITION XXXIII.

THEOREM.

IF the whole hath a greater Reason to the whole, than a part to a part; the Remainder shall have a greater Reason to the Remainder, than the whole to the whole.

13. 4. 6. 2.
A, B, C, D.

If there be a greater Reason of AB to CD, than of B to D; there shall be a greater Reason of A to C, than of AB to CD.

Demonstration. We suppose that there is a greater Reason of AB to CD, than of B to D: thence (*by the 26th.*) there will be a greater Reason of AB, to B, than of CD to D: and (*by the 32d.*) there will be a lesser Reason of AB to CD, than of A to C.

PROPOSITION XXXIV.

THEOREM.

IF there be proposed Two Ranks of Magnitudes; and if the Reason of the first of the first, to the first of the second, be greater than that of the second, to the second; and this greater than that of the Third to the third: there shall be a greater Reason of all the first Ranks, to all the second, than of all the first Rank, excepting the first, to all the second Rank excepting the first, but it shall be less than the first of the first Rank to the first of the second: and in fine greater than the last of the first, to the last of the second.

12. 6. 4	4, 3. 2.
A, B, C	E, F, G,

If there be a greater Reason of A to E, than of B to F; and if the Reason of B to F be greater than that of C to G: I say A, B, C, have a greater Reason to E, F, G, than C to G.

Demonstration. There is a greater Reason of A to E, than of B to F; there shall be also a greater Reason of A to B, than of E to F; and by Composition the Reason

Reaſon of A B to B ſhall be greater than of E F to F; and by exchange, there ſhall be a greater Reaſon of A B to E F, than of B to F. Now the Reaſon of B to F is greater than that of C to G. Thence the Reaſon of A B to E F, is greater than that of C to G: and by compoſition, there ſhall be a greater Reaſon of A, B, C, to E, F, G, than of C to G.

I ſay in the ſecond place, that the Reaſon of A, B, C, to E, F, G, is greater than the Reaſon of B C to F G.

Demonſtration. It is ſuppoſed that there is greater Reaſon of A to E, than of B to F: and by exchange, the Reaſon of A to B, is greater than that of E to F: and by compoſition, there ſhall be a greater Reaſon of A B to B, than of E F to F: and by exchange, the Reaſon of A B to E F ſhall be greater than that of B to F. Moreover, ſeeing there is a greater Reaſon of the whole A B, to E F, than of the part B to F: A ſhall have a greater Reaſon to E, than A B to E F; and there ſhall be a greater Reaſon of B to F, than of B C to F G. And by exchange, there ſhall be a greater Reaſon of A to B C, than of E to F G; and by compoſition, there ſhall be a

greater

greater Reason of A B C, to E F G, than of B C to F G.

I say in the third place that there is a greater Reason of A to B, than of A, B, C, to E F G.

Demonstration. We have already demonstrated that there is a greater Reason of A, B, C, to E, F, G, than of the part B C to the part F G: There shall thence be a greater Reason of A to E, than of A, B, C, to E, F, G, (*by the* 32*d.*)

The end of the Fifth Book.

THE

THE SIXTH BOOK OF Euclid's *Elements*.

THis Book explaineth and beginneth to apply particular matters of the Doctrine of Proportions, which the preceding Book Explaineth but in General. It beginneth with the moſt eaſieſt figures, that is to ſay, Triangles, giving Rules to determine not only the Proportion of their Sides, but alſo that of their capacity, area or ſuperficies. Then it teacheth to find Proportional Lines, and to augmenter diminiſh any figure whatever, according to a given Ratio. It Demonſtrateth the Rule of Three; it applieth the forty ſeventh of the firſt, to all ſorts of Figures. In fine, it giveth us the moſt eaſie and moſt certain principles to conduct us in all ſorts of Meaſuring.

The

The DEFINITIONS.

Def. 1. Fig. I. Plate 6. 1. RIght Lined Figures are like, when that they have all their Angles equal, and the ſides which formeth thoſe Angles Proportional.

Fig. I. *As the Triangles A B C, D E F, ſhall be like, if the Angles A and D; B and E; C and F are equal; and if there be the ſame Ratio of A B to A C, as of D E to D F; and of A B to C B, as of D E to E F.*

Fig. II. 2. Figures are reciprocal, when they may be compared after ſuch ſort, that the Antecedent of one Reaſon, and the Conſequent of the other, be found in the ſame Figure.

That is, when the Anologie beginneth in one figure, and endeth in the ſame. As if there be the ſame Reaſon of A B to C D, as of D E to B F.

Fig. III. 3. A Line is divided into extream and mean Proportion or Reaſon; when there is the ſame Reaſon of the whole Line to its greateſt part, as of its greateſt part to its leſſer part.

As if there be the same Reason of AB to AC, as of AC to CB; the Line AB shall be divided in the point C in extream and mean proportion.

4. The Altitude or height of a figure, is the Length of the Perpendicular drawn from the top thereof to its Base.

As in the Triangles ABC, EFG, the Perpendicular EH, AD; whether it fall without or within the Triangles; is their height, or Altitude. Triangles and Parallelograms, whose heights are equal, may be placed between the same Parallels, for having placed their Bases on the same Line HC; if the Perpendiculars DA, HE, are equal, the Lines EA, HC, shall be parallel. Fig. IV.

5. A Reason is composed of several Reasons, when the Quantities of those Reasons being Multiplied, make a Third Reason.

It is to be taken notice of, that a Reason, at least the rational, hath its name taken from some number which specifieth the Reason or habitude of the Antecedent of that Reason, to its Consequent. As when there is proposed Two Magnitudes or Quantities, the one of Twelve Foot, and the other of Six, we say that the Reason of Twelve to Six is double. In like manner, when there

there is proposed Two Magnitudes, Four and Twelve; we would say it is a subtriple Reason; and $\frac{1}{3}$ is the Denominator thereof, which specifieth that there is the same Reason of Four to Twelve, as of $\frac{1}{3}$ to One, or as of One to Three. One may call this Denominaotr, the Quantity of the Reason. Let there be proposed Three Terms, Twelve, Six, Two: The First Reason of Twelve to Six is double, its Denominator is Two; the Reason of Six to Two is Triple, its Denominator is Three: the Reason of Twelve to Two is compounded of the Reason of Twelve to Six, and of that of Six to Two; to have the Denominator or the Reason of Twelve to Two, which is compounded of Double, and of Triple, multiply Three by Two, and you shall have Six; thence the Reason of Twelve to Two is Sextuple. This is that which Mathematicians understand by composition of Reason, although it ought to be called Multiplication of Reason.

PRO-

PROPOSITION I.

THEOREM.

Parallelograms and Triangles of equal height have the ſame Reaſon as their Baſes.

Let there be propoſed the Triangles Plate VI.
A C G, D E M, of equal height, in ſuch ſort that they may be placed between the ſame Parallels A D, G M: I ſay that there ſhall be the ſame Reaſon of the Baſe G C to the Baſe E M, as of the Triangle A G C, to the Triangle D E M. Let the Baſe E M be divided into as many equal parts as you pleaſe, and let there be drawn to each Diviſion the Lines D F, D H, *&c.* Let alſo the Line G C be divided into parts equal to thoſe of E M, and let be drawn Lines to each diviſion from the top A: All thoſe little Triangles which are made in the Two great Triangles, are between the ſame Parallels, and they have equal Baſes; they are thence equal (*by the* 28*th.*)

Demonſtration. The Baſe G C, contains as many Aliquot parts of the Line E M, as could be found parts equal to E F:

EF: Now as many times as there are in the Base G C parts equal to EF; so many times the Triangle A G C containeth the little Triangles equal to those which are in the Triangle D E M; which being equal among themselves, are its Aliquot parts: thence as many times as the Base G C containeth Aliquot parts of E M, so many times the Triangle A G C containeth Aliquot parts of the Triangle D E M; which will happen in all manner of Divisions. There is thence the same Reason of the Base G C to the Base E M, as of the Triangle A G C to the Triangle D E M.

Coroll. Parallelograms drawn on the same Bases, and that are between the same Parallel Lines, are double to the Triangle (*by the 41st.*) they are thence in the same Reason, as Triangles; that is to say, in the same Reason as their Bases.

USE.

Fig. I. *This Proposition is not only necessary to Demonstrate those which follow, but we may make use thereof in Dividing of Land. Let there be proposed a Trapezium*

pezium A B C D, which hath its ſides A D, B C, parallel, and admit one would cut of a third part: Let C E be made equal to A D; and B G the third part of B E. Draw A G. I ſay that the Triangle A B G is the Third of the Trapezium A B C D.

Demonſtration The Triangles *A D F, F C E, are equiangled, becauſe of the Parallel A D, C E; and they have their Sides A D, C E, equal: They are thence equal* (by the 26th. of the 1ſt.) *and by conſequence the Triangle A B E is equal to the Trapezium. Now the Triangle A B G is the third part of the Triangle A B E by the preceding Propoſition; thence the Triangle A B G is the third part of the Trapezium A B C D.*

PROPOSITION II.

THEOREM.

A Line being drawn in a Triangle Parallel to its Baſe, divideth its ſides proportionally, and if a Line divideth proportionally the ſides of a Triangle, it ſhall be parallel to its Baſe.

If in the Triangle A B C, the Line D E,

DE, is Parallel to the Baſe BC; the Sides AB, AC, ſhall be divided proportionally; that is to ſay, that there ſhall be the ſame Reaſon of AD to DB, as of AE to EC. Draw the Lines DE, BE. The Triangles DBE, DEC, which have the ſame Baſe DE, and are between the ſame Parallels DE, BC; are equal (*by the 37th. of the 1ſt.*)

Demonſtration. The Triangles ADE, DBE, have the ſame point E for their vertical, if we take AD, DB, for their Baſes; and if one ſhould draw through the point E, a Parallel to AB, they would be both between the ſame Parallels; they ſhall have thence the ſame Reaſon as their Baſes (*by the 1ſt.*) that is to ſay, that there is the ſame Reaſon of AD to DB, as of the Triangle ADE to the Triangle DBE, or to its equal CED. Now there is the ſame Reaſon of the Triangle ADE to the Triangle CED, as of the Baſe AE to EC. There is therefore the ſame Reaſon of AD to DB, as of AE to EC.

And if there be the ſame Reaſon of AE to EC, as of AD to DB: I ſay that the Lines DE, BC, would then be Parallels.

Demonſtration.

Demonſtration. There is the ſame Reaſon of A D to D B, as of the Triangle A D E to the Triangle D B E (*by the 1ſt.*) there is alſo the ſame Reaſon of A E to E C, as of the Triangle A D E to the Triangle D E C; conſequently there is the ſame Reaſon of the Triangle A D E to the Triangle B D E, as of the ſame Triangle A D E to the Triangle C E D. So then (*by the 7th. of the 5th.*) the Triangles B D E, C E D, are equal: And (*by the 39th. of the 1ſt.*) they are between the ſame Parallels.

U S E

THis Propoſition is abſolutely neceſſary, in the following Propoſitions, one may make uſe thereof in Meaſuring as in the following figure: If it were required to meaſure the height B E; having the length of the ſtaff D A, there is the ſame Reaſon of C D to D A, as of B C to B E.

PROPOSITION III.

THEOREM.

THat Line which divideth the Angle of a Triangle, into two equal parts, divideth its Baſe in two parts, which are in the ſame Reaſon to each other as are their Sides. And if that Line divideth the Baſe into parts proportional to the Sides, it ſhall divide the Angle into Two equally.

If the Line AD divideth the Angle BAC into Two equal parts; there ſhall be the ſame Reaſon of AB to AC, as of BD to DC. Continue the Side CA, and make AE equal to AB; then draw the Line EB.

Demonſtration. The exterior Angle CAB is equal to the Two interior Angles AEB, ABE; which being equal (*by the* 5*th. of the* 1*ſt.*) ſeeing the Sides AE, AB, are equal; the Angle BAD, the half of BAC, ſhall be equal to one of them; that is to ſay to the Angle ABE. Thence (*by the* 27*th. of the* 1*ſt.*) the Lines AD, EB, are parallel; and (*by the* 2*d.*) there is the ſame Reaſon of EA, or AB to AC, as of BD to DC.

Secondly.

Secondly. If there be the ſame Reaſon of A B to A C, as of B D to D C, the Angle B A C ſhall be divided into Two equally.

Demonſtra. There is the ſame reaſon of A B or A E to A C, as of B D to D C: thence the Lines E B, A D, are parallel; and (*by the 29th. of the 1ſt.*) the Alternate Angles E B A, B A D, the internal B E A, and the external D A C, ſhall be equal; and the Angles E B A, A E B, being equal; the Angles B A D, D A C, ſhall be ſo likewiſe. Wherefore the Angle B A C hath been divided equally.

USE.

WE make uſe of this Propoſition to attain to the Proportion of the ſides.

PROPOSITION IV.

THEOREM.

EQuiangular Triangles have their Sides Proportional.

If the Triangles A B C, D C E, are equiangular; that is to ſay, that the

Angles A B C, D C E; B A C, C D E, be equal: There will be the ſame Reaſon of B A to B C, as of C D to C E. In like manner the reaſon of B A to A C, ſhall be the ſame with that of C D to D E. Joyn the Triangles after ſuch a manner, that their Baſes B C, C E, be on the ſame Line; and continue the ſides E D, B A: ſeeing the Angles A C B, D E C, are equal; the Lines A C, E F, are parallel; and ſo C D, B F, (*by the 29th. of the 1ſt.*) and A F, D C, ſhall be a parallelogram.

Demonſtration. In the Triangle B F E, A C, is parallel to the Baſe F E; thence (*by the 2d.*) there ſhall be the ſame reaſon of B A to A F, or C D, as of B C to C E; (*and by exchange*) there ſhall be the ſame reaſon of A B to B C, as of D C to C E. In like manner in the ſame Triangle, C D being parallel to the Baſe B F; there ſhall be the ſame Reaſon of F D, or A C to D E, as of B C to G E (*by the 2d.*) and by exchange, there ſhall be the ſame reaſon of A C to B C, as of D E to C E.

USE.

USE.

THis Propoſition is of a great extent, and may paſs for a univerſal Principle in all ſorts of Meaſuring. For in the firſt place the ordinary practice in meaſuring inacceſſible Lines, by making a little Triangle like unto that which is made or imagined to be made on the ground, is founded on this Propoſition, as alſo the greateſt part of thoſe Inſtruments, on which are made Triangles like unto thoſe that we would meaſure, as the Geometrical Square, Sinical Quadrant, Jacobs Staff, and others. Moreover we could not take the plane of a place, but by this Propoſition: wherefore to explain its uſes, we ſhould be forced to bring in the firſt Book of practical Geometry.

PROPOSITION V.

THEOREM.

TRiangles whoſe ſides are proportional, are equiangular.

If the Triangles ABC, DEF, have their ſides proportional, that is to ſay, if there be the ſame reaſon of AB to

BC, as of DE to EF; as alſo if there be the ſame reaſon of AB to AC, as of DE to DF, the Angles ABC, DEF, A and D; C and F ſhall be equal. Make the Angle FEG equal to the Angle B; and EFG equal to the Angle C.

Demonſtration. The Triangles ABC, EFG, have two Angles equal; they are thence equiangled (*by the Cor. of the 32d. of the 1ſt.*) and (*by the 4th.*) there is the ſame reaſon of DE to EF, as of EG to EF. Now it is ſuppoſed that there is the ſame reaſon of DE to EF, as of EG to EF. Thence (*by the 7th. of the 5th.*) DE, EG, are equal. In like manner DF, FG, are alſo equal, and (*by the 8th. of the 1ſt.*) the Triangles DEF, GEF, are equiangular. Now the Angle GEF was made equal to the Angle B: thence DEF is equal to the Angle B; and the Angle DFE, to the Angle C. So that the Triangles ABC, DEF, are equiangular.

PROPOSITION VI.

THEOREM.

TRiangles which have their ſides proportional, which include an equal Angle, are equiangular.

If

If the Angles B and E of the Triangles ABC, DEF, being equal, there be the ſame reaſon of AB to BC, as of DE to EF; the Triangles ABC, DEF, ſhall be equiangular. Make the Angle FEG, equal to the Angle B, and the Angle EFG, equal to the Angle C.

Demonſtra. The Triangles ABC, EGF, are equiangular (*by the Cor. of the 32d. of the 1ſt.*) there is thence the ſame reaſon of AB, to BC, as of EG to EF, (*by the 4th.*) Now as AB to BC, ſo is DE to EF; there is then the ſame reaſon of DE to EF, as of GE to EF. So then (*by the 7th of the 5th*) DE, EG, are equal; and the Triangles DEF, GEF, which have their Angles DEF, GEF, each of them equal to the Angle B, and the Sides DE, EG, equal, with the Side EF common; they ſhall be equal in every reſpect (*by the 4th. of the 1ſt.*) they are thence equiangular; and the Triangle EGF being equal to the Triangle ABC, the Triangles ABC, DEF, are equiangular.

The Seventh Propoſition is unneceſſary.

PROPOSITION VIII.

THEOREM.

A Perpendicular being drawn from the Right Angle of a Right Angled Triangle to the oppoſite ſide, divideth the ſame into Two Triangles which are alike thereto.

If from the Right Angle ABC be drawn a perpendicular BD to the oppoſite ſide AC, it will divide the Right Angled Triangle ABC into Two Triangles ADB, BDC, which ſhall be like, or equiangular to the Triangle ABC.

Demonſtration. The Triangles ABC, ADB, have the ſame Angle A; the Angle ADB, ABC, are right: they are thence equiangular (*by the Cor. 2. of the 32d. of the 1ſt.*) In like manner the Triangles BDC, ABC, have the Angle C common; and the Angles ABC, BDC, being right, they are alſo equal. Thence the Triangles ABC, DBC, are like.

USE.

We meaſure inacceſſable diſtances by a Square, according to this Propoſition.

Propoſition. For example, if we would meaſure the diſtance DC, having drawn the perpendicular DB, and having put a Square at the Point B, in ſuch manner, that by looking over one of its Sides BC, I ſee the Point C, and over its other Side I ſee the Point A; it is evident that there will be the ſame reaſon of AD to DB, as of DB to DC. So that multiplying DB by its ſelf, and dividing that product by AD, the Quotient ſhall be DC.

PROPOSITION IX.

PROBLEM.

To cut off from a Line any part required.

LEt there be propoſed the Line AB, from which it is required to cut off three Fifths. Make the Angle ECD at diſcretion, take in one of thoſe Lines CD, five equal parts; and let CF be three of the ſame, and CE be equal to AB. Then draw the Line DE; after which draw FG parallel to DE; the Line CG will contain three Fifths of CE or AB.

Demonſtration. In the Triangle ECD, FG being parallel to the Baſe DE, there will be the ſame Reaſon of CF to FD, as of CG to GE (*by the ſecond.*) and by compoſition (*by the 18th. of the 5th.*) there ſhall be the ſame Reaſon of CG to CE, as of CF to CD. Now CF contains three fifths of CD: wherefore CG ſhall contain three fifths of CE, or AB.

PROPOSITION X.

PROBLEM.

TO divide a Line after the ſame manner as another Line is divided.

If one would divide the Line AB after the ſame manner as the Line AC is divided: Joyn thoſe Lines, making an Angle at pleaſure, as CAB: Draw the Line BC, and the parallels EO, FV, and the Line AB ſhall be divided after the ſame manner as AC.

Demonſtration. Seeing that in the Triangle BAC, the Line HX hath been drawn parallel to the Baſe BC; it will divide the Sides AB, AC, proportionally (*by the ſecond*) it is the ſame with all the other parallels.

To

To do the ſame with more facility, one may draw B D parallel to A C, and put off the ſame Diviſions of A C on B D; then draw the Lines from the one to the other.

PROPOSITION XI.

THEOREM.

TO find a third Proportional to Two given Lines.

It is required to find a third proportional to the Lines A B, B C, that is to ſay, that there may be the ſame reaſon of A B to B C, as of B C to the Line required. Make at diſcretion the Angle E A C; put off one after the other, the Lines A B, B C; and let A D be equal to B C: Draw the Lines B D, and its parallel C E. The Line D E ſhall be that which you require.

Demonſtration. In the Triangle E A C, the Line D B is parallel to the Baſe C E: There is thence (*by the* 2*d.*) the ſame reaſon of A B to B C, as of A D, or B C to D E.

PRO-

PROPOSITION XII.

PROBLEM.

TO find a fourth Proportional to three Lines given.

Let there be propoſed three Lines AB, BC, DE, to which muſt be found a fourth proportional, make an Angle as FAC, at diſcretion; take on AC the Lines AB, BC; and on AF, the Line AD equal to DE: then draw DB, and its parallel FC. I ſay that DF is the Line you ſeek for; that is to ſay, that there is the ſame Reaſon of AB to BC, as of DE, or AD to DF.

Demonſtration. In the Triangle FAC, the Line DB is parallel to the Baſe FC; there is thence the ſame reaſon of AB to BC, as of AD to DF (*by the 2d.*)

USE.

THe uſe of the Compaſs of Proportion (or Sector) is eſtabliſhed on theſe Propoſitions: for we divide a Line as we pleaſe,

pleaſe, by the Compaſs of Proportion: we do the Rule of Three without making uſe of Arithmetick: we extract the Square Root and Cube Root: we double the Cube: we meaſure all ſorts of Triangles: we find the Content of Superficies, and the ſolidity of Bodies; we augment or diminiſh any figure whatever, according to what Proportion we pleaſe; and all thoſe uſes are Demonſtrated by the foregoing Propoſitions.

PROPOSITION XIII.

PROBLEM.

TO find a mean Proportional between Two Lines.

If you would have a mean Proportional between the Lines L V, V R: having joyned them together on a ſtrait Line, divide the Line L R into two equal parts in the point M; and having deſcribed a Semi-circle L T R on the Center M; draw the perpendicular V T, it ſhall be a mean Proportional between L V, V R. Draw the Lines L T, T R.

Demonſtration. The Angle L T R, deſcribed in a Semi-circle, is right (*by the* 31*ſt. of the* 3*d.*) and (*by the* 8*th.*) the Triangles L V T, T V R, are like; there

is

is thence the ſame Reaſon in the Triangle L V T, of L V to V T, as of V T to V R in the Triangle T V R, (*by the 4th.*) So then V T is a mean Proportional between L V and V R.

USE.

WE Reduce to a Square any Rectangular Parallelogram whatever by this Propoſition. For example, in the Rectangle comprehended under L V, V R, I will demonſtrate hereafter, that the Square of V T is equal to a Rectangle comprehended under L V, and V R.

PROPOSITION XIV.

THEOREM.

EQuiangular and equal Parallelograms have their Sides reciprocal and equiangular Parallelograms whoſe ſides are reciprocal, are equal.

If the Parallelograms L and M be equiangular and equal, they ſhall have their ſides reciprocal; that is to ſay, that there ſhall be the ſame Reaſon of C D to D E, as of F D to D B. For ſeeing they have their Angles equal, they may be joyned after ſuch manner that their

joyned

ſides C D, D E, be on one ſtreight Line (*by the* 15*th. of the* 1*ſt.*) continue the Sides A B, G E; you will have compleated the Parallelogram B D E H.

Demouſtration. Seeing the Parallelogram L and M are equal, they ſhall have the ſame reaſon to the Parallelogram B D E H. Now the reaſon of the parallelogram L, to the Parallelogram B D E H, is the ſame with that of the Baſe C D to the Baſe D E (*by the* 1*ſt.*) and that of the Parallelogram M, or D F G E, is the ſame with that of the Baſe F D to the Baſe B D. Thence there is the ſame reaſon of C D to D E, as of F D to B D.

Secondly, if the equiangular Parallelograms L and M, have their Sides reciprocal, they ſhall be equal.

Demonſtration. The Sides of the Parallelograms are reciprocal; that is to ſay, that there is the ſame Reaſon of C D to D E, as of F D to B D: Now as the Baſe C D is to D E, ſo is the parallelogram L to the parallelogram B D E H, (*by the firſt,*) and as F D is to D B, ſo is the parallelogram M to B E D H; there is thence the ſame Reaſon of L to B D E H, as of M to the ſame B D E H, ſo then (*by the* 7*th. of*

the

the 5th.) the parallelograms L and M are equal.

PROPOSITION XV.

THEOREM.

EQual Triangles which have one Angle equal, have the Sides which form that Angle reciprocal; and if their ſides be reciprocal, they ſhall be equal.

If the Triangles F and G, being equal, have their Angles A C B, E C D, equal: their ſides about that Angle ſhall be reciprocal; that is to ſay, that there ſhall be the ſame Ratio of B C to C E, as of C D to C A. Diſpoſe the Triangles after ſuch a manner, that the Sides C D, C A, be one ſtraight Line: ſeeing the Angles A C B, E C D, are ſuppoſed equal, the Lines B C, C E, ſhall be alſo a ſtraight Line, (*by the 14th. of the 1ſt.*) Draw the Line A E.

Demonſtration. There is the ſame Ratio of the Triangle A B C to the Triangle A C E, as of the Triangle E C D, equal to the firſt, to the ſame Triangle A C E, (*by the 7th. of the 5th.*) Now as A B C, is to A C E, ſo is the Baſe B C

to

to the Baſe CE, (*by the 1ſt.*) ſeeing they have the ſame vertical A; and as FCD is to ACE, ſo is the Baſe CD to CA. Now if it be ſuppoſed that the ſides are reciprocal; that is to ſay, that there be the ſame Ratio of BC to CE, as of CD to CA; the Triangles ABC, CDE, ſhall be equal, becauſe they would then have the ſame Ratio to the Triangle ACE.

PROPOSITION XVI.

THEOREM.

IF four Lines be proportional, the Rectangle comprehended under the firſt and fourth, is equal to the Rectangle comprehended under the ſecond and third, and if the Rectangle comprehended under the extremes, be equal to the Rectangle comprehended under the means, then are the four Lines proportional.

If the Lines AB, CD, be proportional, that is to ſay, that there is the ſame Ratio of A to B, as of C to D; the Rectangle comprehended under the firſt A, and the fourth D, ſhall be equal to the Rectangle comprended under B and C.

Demonſtration.

Demonſtration. The Rectangles have their Angles equal, ſeeing it is Right; they have alſo their Sides reciprocal; they are thence equal (*by the* 16*th.*)

In like manner, if they be equal, their ſides are reciprocal; that is to ſay, that there is the ſame Ratio of A to B, as of C to D.

PROPOSITION XVII.

THEOREM.

IF three Lines be proportional, the Rectangle comprehended under the firſt and third is equal to the Square of the mean, and if the Square of the mean be equal to the Rectangle of the extrems, the three Lines are then proportional.

If the three Lines A, B, D, be proportional; the Rectangle comprehended under A, and under D, ſhall be equal to the Square of B. Take C equal to B; there ſhall be the ſame Ratio of A to B, as of C to D: thence the four Lines A, B, C, D, are proportional.

Demonſtration. The Rectangle under A and D ſhall be equal to the Rectangle under B and C (*by the foregoing*:) Now this laſt Rectangle is a Square, ſeeing the

the Lines B, and C, are equal: thence the Rectangle under A and D is equal to the Square of B.

In like manner, if the Rectangle under A and D be equal to the Square of B, there ſhall be the ſame Ratio of A to B, as of C to D; and ſeeing that B and C are equal, there ſhall be the ſame Ratio of A to B, as of B to D.

USE.

THoſe Four Propoſitions demonſtrateth that Rule of Arithmetick which we commonly call the Rule of Three, and conſequently the Rule of Fellowſhip, falſe poſition, and all other which are performed by Proportion. For example, let there be propoſed theſe three Numbers, A Eight, B ſix, C four, and it is required to find the fourth proportional. Suppoſe it to be found, and let it be D. The Rectangle comprehended under A and D is equal to to the Rectangle comprehended under B and C. Now I can have this Rectangle by Multiplying B by C; that is to ſay, ſix by four, and I ſhall have twenty four, thence the Rectangle comprehended under A and D, is twenty four, wherefore dividing the ſame by A eight,

eight, the Quotient three is the number I look for.

PROPOSITION XVIII.

THEOREM.

TO describe a Poligon like to another on a Line given.

There is propoſed the Line A B, on which one would deſcribe a Poligon like unto the Poligon C F D E. Having divided the Poligon C F D E into Triangles, make on the Line A B a Triangle A B H like unto the Triangle C F E; that is to ſay, make the Angle A B H equal to the Angle C F E, and B A H equal to F C E. So then the Triangles A B H, C F E, ſhall be equiangled (*by the 32d. of the firſt.*) make alſo on B H, a Triangle equiangled to F D E.

Demonſtration. Seeing the Triangles which are parts of the Poligons, are equiangular, the two Poligons are equiangular. Moreover, ſeeing the Triangles A B H, C F E, are equiangular, there is the ſame Ratio of A B to B H, as of C F to F E, (*by the 4th.*) In like manner, the Triangles H B G, E F D, being equiangular, there ſhall be the ſame

ſame Ratio of BH to BG, as of FE to FD; and by equality there ſhall be the ſame Ratio of AB to BG, as of CF to FD. And ſo of the reſt of the ſides. Thence (*by the firſt Definition,*) the Poligons are like to each other.

USE.

IT is on this Propoſition we eſtabliſh the greateſt part of the practical ways to take the plane of a place, of an Edifice, of a Field, of a Forreſt, or of a whole Country; for making uſe of the equal parts of a Line for Feet or for Chains; we deſcribe a figure like unto the Prototype, but leſſer, in which we may ſee the Proportion of all its Lines. And becauſe it is eaſier on paper than on the ground; we may comprehend in this Propoſition all Geodeſ, all Chorography, all Geographical Charts, and ways of reducing of the greater into a leſſer; wherefore this Propoſition extends almoſt to all Arts, in which it is neceſſary to take a deſign or Model.

PRO-

PROPOSITION XIX.

THEOREM.

Like Triangles are in duplicate Ratio to their homologous sides.

If the Triangles ABC, DEF, be like or equiangular; they shall be in duplicate Ratio of their homologous Sides BC, EF; that is to say, that the Ratio of the Triangle ABC, to the Triangle DEF shall be in duplicate Ratio of BC to EF; wherefore, by seeking the third Proportional HI to the Lines BC, EF; or so making it, that there may be the same Ratio of BC to EF, as of EF to HI; the Triangle ABC shall have the same Ratio to the Triangle DEF, as the Line BC hath to the Line HI. Which is called a duplicate Ratio, or doubled Reason (*by the 11th. Def. of the fifth.*) Let BG and HI be equal; and let the Line AG be drawn.

Demonstration. The Angle B and E of the Triangles ABG, DEF, are equal: also seeing the Triangle ABC, DEF, are like; there shall be the same Ratio (*by the 4th.*) of AB to DE, as

of BC to EF: Now as BC is to EF, ſo is EF to HI, or BG: thence as AB is to DE, ſo is EF to BG: and conſequently the ſides of the Triangles ABG, DEF, being reciprocal, the Triangles ſhall be equal (*by the* 15*th.*) Now (*by the firſt,*) the Triangle ABC hath the ſame Ratio to the Triangle ABG, as BC to BG, or HI: thence the Triangle ABC hath the ſame Ratio to the Triangle DEF, as BC to HI.

USE.

THOSE Propoſitions do correct the opinions of many, who eaſily imagine, that like figures have the ſame Ratio as their Sides. For example, let there be propoſed two Squares, Two Pentagons, two Hexagons, two Circles, and let the ſides of the firſt be double to the ſide of the ſecond; the firſt figure ſhall be quadruple to the ſecond. If the ſide of the firſt be triple to that of the ſecond, the firſt figure ſhall be nine times greater than the ſecond. So that to find a Square triple to another, there muſt be ſought a mean Proportional between one and three, which would be almoſt 1 $\frac{3}{4}$ *for the ſide of the tripple figure.*

PRO-

PROPOSITION XX.

THEOREM.

Like Poligons may be divided into ſo many like Triangles, and they are in duplicate Ratio to their homologous ſides.

If the Poligons ABCDE, GHIML, be like; they may be divided into ſo many like Triangles, and which ſhall be like parts of their whole. Draw the Lines AC, AD, GI, GL.

Demonſtration. Seeing the Poligons are alike, their Angles B and H ſhall be equal, and there ſhall be the ſame Ratio of AB to BC, as of GH to HI, (*by the 15th.*) thence the Triangles ABC, GHI are alike: and (*by the 4th.*) there ſhall be the ſame Ratio of BC to CA, as of HI to GI. Moreover, ſeeng there is the ſame Ratio of CD to BC, as of IL to IH; and the ſame Ratio of BC to CA, as of HI to GI. There ſhall be by equality the ſame Ratio of CD to CA, as of IL to GI. Now the Angles BCD, and HIL, being equal, if you take away the equal Angles ACB, GIH, the Angles ACD, GIL, ſhall be equal. Thence the Triangles ACD GIL,

G I L, ſhall be like (*by the* 15*th.*) So that it is eaſie after the ſame manner to go round about the Angles of the Polygons, and to prove they are circular or alike.

I further add, that the Triangles are in the ſame Ratio as are the Polygons.

Demonſtration. Seeing all the Triangles are like, their ſides ſhall be in the ſame Ratio (*by the* 4*th.*) Now each Triangle is to its like, in duplicate Ratio of its homologous ſides (*by the* 19*th.*) thence each Triangle of one Polygon, to each Triangle of the other Polygon, is in duplicate Ratio to the ſides; which being the ſame; there ſhall be the ſame Ratio of each Triangle to its like, as of all the Triangles of one Polygon, to all the Triangles of the other Polygon (*by the* 12*th. of the* 5*th.*) that is to ſay, as of the one Poligon to the other.

Coroll. 1. Like Polygons are in duplicate Ratio to their ſides.

Coroll. 2. If three Lines are continually Proportional, the Polygon deſcribed on the firſt, ſhall have the ſame Ratio to the Polygon deſcribed on the ſecond, as the firſt hath to the third; that is to ſay, in duplicate Ratio of that of the firſt Line to the ſecond.

USE from the Learned Dr. *Barrow.*

BY this is found a method of enlarging or diminiſhing a Right Lined figure in a Ratio given, as if you would have a Pentagon Quintuple of that Pentagon whereof C D is the ſide; then betwixt A B and 5 A B, find out a mean Proportional; upon this raiſe a Pentagon like unto that given, and it ſhall be quintuple alſo of the Pentagon given.

Hence alſo, if the Homologous ſides of like figures be known, then will the Proportion of the figure be evident, viz. *by finding out a third Proportional.*

PROPOSITION XXI.

THEOREM.

POlygons which are like to a Third Polygon, are alſo like one to the other.

If Two Polygons are like to a third, they ſhall be like one to the other; for each of them may be divided into as many like Triangles as there is in the third. Now the Triangles which are like unto the ſame Third, are alſo like one to the other, becauſe the Angles which are equal

equal to a Third, are equal one to the other; and the Angles of the Triangles being equal, those of the Polygons of whom it is compounded, are so likewise.

I farther add that the sides of Triangles being in the same Ratio, those of the Polygons shall be so likewise, seeing they are the same.

PROPOSITION XXII.

THEOREM.

Like Polygons described on four Lines which are Proportional, are also Proportional. And if the Polygons be in the same Ratio, the Lines on which they are described shall be so also.

If there be the same Ratio of B C to E F, as of H T to M N; there shall also be the same Ratio of the Polygon A B C to the like Polygon D E F, as of the Polygon H L to the like Polygon M O. Seek to the Lines BC, E F, a third Proportional G; and to the Lines H T, M N, a third Proportional P (*by the* 11 *th.*) seeing there is the same Ratio of B C to E F, as of H T to M N; and

of E F to G, as of M N to P: there ſhall be by equality the ſame Ratio of B C to G, as of H T to P; and this Ratio ſhall be duplicate or doubled of the Ratio of B C to E F, or of H T to M N.

Demonſtration. The Polygon A B C to the Polygon D E F, is in duplicate or doubled Ratio of the Ratio of B C to E F (*by the* 21*ſt.*) that is to ſay, as B C is to G; and the Polygon H L to M O, hath the ſame Ratio as H T to P. There is therefore the ſame Ratio of A B C to D E F, as of H L to M O.

And if like Polygons are proportional; the Lines being in ſub-duplicate Ratio, ſhall be alſo Proportional.

U S E.

A, B, C, D.
3. 2. 6. 4.
9. 4. 36. 16
E, F, G, H.

This Propoſition may be eaſily applied to Numbers. If the Numbers A, B, C, D, are Proportional, their Squares E F, G H, ſhall be ſo likewiſe; which we make uſe of in Arithmetick, and yet more uſe thereof in Algebra.

PRO-

PROPOSITION XXIII.

THEOREM.

Equiangular Parallelograms have their Ratio compounded of the Ratio of their ſides.

If the Parallelograms L and M be equiangular; the Ratio of L to M ſhall be compounded of that of A B to D E, and that of D B to D F. Joyn the Parallelograms in ſuch a manner that their ſides B D, D F, be on a ſtreight Line, as alſo C D, D E; which may be, if they be equiangular. Compleat the Parallelogram B D E H.

Demonſtration. The Parallelogram L hath the ſame Ratio to the Parallelogram B D E H, as the Baſe A B hath to the Baſe B H or D E (*by the firſt.*) the Parallelogram B D E H hath the ſame Ratio to the Parallelogram D F G E, that is to ſay M, as the Baſe B D hath to the Baſe D F. Now the Ratio of the Parallelogram L to the Parallelogram M, is compounded of that of L to the Parallelogram B D E H, and that of B D E H to the Parallelogram M. Thence the

Ratio of L to M, is compounded of that of AB to DE, and of that of BD to EG. For example, if AB be eight, and BH five, BD four, DF seven; say as four is to seven, so is five to eight $\frac{1}{4}$; you shall have these three numbers eight, five, eight $\frac{1}{4}$, eight to five shall be the Ratio of the Parallelogram L to BDEH, the same as that of AB to DE; five to eight $\frac{1}{4}$ shall be that of the Parallelogram BDEH to M. So then taking away the middle term which is five, you shall have the Ratio of eight to eight $_{4}$, for the Ratio compounded of both; or as 4 times 8 or 32, to 5 times 7 or 35.

PROPOSITION XXIV.

THEOREM.

IN all sorts of Parallelograms; those through which the Diameter passeth, are like to the greater.

Let the Diameter of the Parallelogram AC pass through the Parallelograms EF, GH: I say they are like unto the Parallelogram AC.

Demonstration. The Parallelograms AC, EF, have the same Angle B; and because in the Triangles BCD, IF, is Parallel

Parallel to the Base D C, the Triangles B F I, B C D, are equiangular. There is therefore (*by the 4th.*) the same Ratio of B C to C D, as of B F to F I; and consequently the sides are in the same Ratio. In like manner, I H is Parallel to B C; there shall then be the same Ratio of D H to H I, as of D C to B C; and the Angles are also equal, all the sides being Parallel: thence (*by the 8th. Def.*) the Parallelograms E F, G H, are like unto the Parallelogram A C.

USE.

I Made use of this Proposition in the Tenth Proposition of the last Book of Perspective, to shew that an Image was drawn like unto the Original, with the help of a Parallelogram Composed of four Lines.

PROPOSITION XXV.

PROBLEM.

TO describe a Polygon like unto a given Polygon, and equal to any other Right Lined figure.

If you would describe a Polygon equal

to the Right Lined figure A, and like unto the Polygon B; make a Parallelogram C E equal to the Polygon B, (*by the* 34*th. of the first.*) and on D E, make a Parallelogram E F equal to the Right Lined figure A, (*by the* 45*th. of the first.*) Then seek a mean Proportional G H, between C D and D F (*by the* 13*th.*) Lastly, make on G H a Polygon O, like unto B (*by the* 18*th.*) It shall be equal to the Right Lined figure A.

Demonstration. Seeing that C D, G H, D F, are continually Proportional, the Right Lined figure B described on the first, shall be to the Right Lined figure O described on the second, as C D is to D F, (*by the Coroll. of the* 20*th.*) Now as C D is to D F, so is the Parallelogram C E to the Parallelogram E F, or as B to A, seeing they are equal. There is thence the same Ratio of B to O, as of B to A. So then (*by the* 7*th. of the* 5*th.*) A and O are equal.

USE.

This Proposition contains a change of figures, keeping always the same Area, which is very useful, principally in Practical Geometry, to reduce an irregular figure into a Square.

PROPOSITION XXVI.

THEOREM.

IF in one of the Angles of a Parallelogram, there be described a leſſer Parallelogram like unto the greater, the Diameter of the greater ſhall meet the Angle of the leſſer.

If in the Angle D, of the Parallelogram A C; be deſcribed another leſſer D G, like thereto: The Diameter D B ſhall paſs through the point G. For if it did not, but it paſſed through I, as doth the Line B I D. Draw the Line I E Parallel to H D.

Demonſtration. The Parallelogram D I, is like unto the Parallelogram A C, (*by the 24th.*) Now it is ſuppoſed that the Parallelogram D G is alſo like thereto; thence the Parallelograms D I, D G, would be like; which is impoſſible: otherwiſe, there would be the ſame Ratio of H I to I E or G F, as of H G to G F; and (*by the 7th. of the 5th.*) the Lines H I, H G, would be equal.

The Twenty Seventh, Twenty Eighth, and Twenty Ninth Propoſitions are unneceſſary.

PROPOSITION XXX.

PROBLEM.

TO cut a Line given into extream and mean Proportion.

It is propoſed to cut the Line A B in extream, and mean Proportion; that is to ſay, in ſuch a manner, that there may be the ſame Ratio of A B to A C, as of A C to C B. Divide the Line A B (*by the* 11*th. of the ſecond*) in ſuch a manner that the Rectangle comprehended under A B, C B, be equal to the Square of A C.

Demonſtration. Seeing the Rectangle of A B, B C, is equal to the Square of A C; there will be the ſame Ratio of A B to A C, as of A C to B C (*by the* 17*th.*)

USE.

THis Propoſition is neceſſary in the Thirteenth Book of Euclid, *to find the length of the Sides of ſome of the five Regular Bodies. Father* Lucas *of St.* Sepulchres

pulchres *hath Compoſed a Book of the Proprieties of a Line, which is cut into extreams and mean Proportion.*

PROPOSITION XXXI.

THEOREM.

A Polygon which is deſcribed on the Baſe of a Right Angled Triangle, is equal to the other Two like Polygon deſcribed on the other Sides of the ſame Triangle.

If the Triangle A B C hath a Right Angle B A C; the Polygon D deſcribed on the Baſe B C, is equal to the like Polygons F, and E, deſcribed on the Sides A B, A C.

Demonſtration. The Polygons D, E, F, are amongſt themſelves in duplicate Ratio of their homologous Sides B C, A C, A B, (*by the 20th.*) If there were deſcribed a Square on thoſe ſides, they would be alſo in duplicate Ratio to their ſides. Now (*by the 47th. of the firſt.*) the Square of B C would be equal to the Squares of A C, A B; thence the Polygon D deſcribed on B C, is equal to the like Polygons E and F, deſcribed on A B, A C.

USE.

This Proposition is made use of to make all sorts of figures greater or lesser, for it is more universal then (the 47th. of the 1st.) *which notwithstanding is so useful, that it seemeth that almost all Geometry is established on this Principle.*

A

The 32d. Proposition is unnecessary.

PROPOSITION XXVIII.

THEOREM.

IN equal Circles, the Angles as well those at the Center as those at the Circumference, as also the Sectors are in the same Ratio as the Arks, which serve to them as Base.

If the Circles A N C, D Q F, be equal; there shall be the same Ratio of the Angle A B C to the Angle D E F, as of the Ark A C to the Ark D F. Let the Ark A G, G H, H C, be equal Arks, and consequently Aliquot parts of the Ark A C, and let be divided

the

the Ark DE, into ſo many equal to AG, as may be found therein, and let the Lines EH, EK, and the reſt be drawn.

Demonſtration. All the Angles ABG, GBH, HBC, DEI, IEK, and the reſt, are equal (*by the 27th. of the third*;) ſo then AG an Aliquot part of AC, is found in the Ark DF, as many times as the Angle ABG an Aliquot part of the Angle ABC, is found in DEE; there is therefore the ſame Ratio of the Ark AC to the Ark DF, as of the Angle ABC to the Angle DEF. And becauſe N and O are the halfs of the Angles ABC, DEF, they ſhall be in the ſame Ratio as are thoſe Angles; there is therefore the ſame Ratio of the Angle N to the Angle O, as of the Ark AC to the Ark DF.

It is the ſame with Sectors; for if the Lines AG, GH, HC, DI, IK, and the reſt were drawn, they would be equal (*by the 28th. of the Third*;) and each Sector would be divided into a Triangle and a Segment. The Triangles would be equal (*by the 8th. of the firſt*;) and the little Segments would be alſo equal (*by the 24th. of the third*;) thence all thoſe little

Sectors would be equal: and ſo as many as the Ark BF containeth of Aliquot parts of the Ark of AG, ſo many the Sector DKF would contain Aliquot parts of the Sector AGC. There is therefore the ſame Reaſon of Ark to Ark, as of Sector to Sector.

The end of the Sixth Book.

THE

THE ELEVENTH BOOK OF Euclid's *Elements*.

THis Book comprehendeth the first Principles of Solid Bodies, whence it is impossible to establish any thing touching the third species of quantity, without knowing what this Book teacheth us, which maketh it very necessary in the greatest part of the Mathematicks, in the first place, Theodosius's *Spherick, doth wholly suppose it, Spherical Trigonometry, the third part of Practical Geometry, several Propositions of Staticks, and of Geography, are established on the principles of Solid Bodies, Gnomonicks, Conical Sections, and the Treatise of Stone-cutting, are not difficult, but because one is often obliged to represent on paper, figures that are raised, and which are comprised under*

under several Surfaces. I leave out the Seventh, *the* Eighth, *the* Ninth, *and the Tenth Books of* Euclid's Elements, *because they are unnecessary, in almost all the parts of the Mathematicks. I have often wondred that they have been put amongst the* Elements, *seeing it is evident that* Euclid *did Compose them only to establish the Doctrine of incommensurables, which being only a curiosity, ought not to have been placed with the Books of Elements, but ought to make a particular Treatise. The same may be said of the* Thirteenth, *and of the rest; for I believe that one may learn almost all parts of the Mathematicks, provided one understands well these* Eight Books *of* Euclid's Elements.

DEFINITIONS.

Def. 1. Plate VII. Fig. I.

1. A Solid is a Magnitude which hath length, breadth and thickness. *As the figure L T; its Length is N X, its Breadth N O, its Thickness L N.*

2. The extremities or terms of a Solid, are Superficies.

3. A Line is at Right Angles, or Perpendicular to a Plane, when it is Perpendicular to all the Lines that it meeteth with in the Plane.

As

As the Line AB shall be at Right Angles to the Plane CD, if it be Perpendicular to the Lines CD, FE; which Lines being drawn in the Plane CD, pass through the Point B, in such sort that the Angles ABC, ABD, ABE, ABF, be Right. Fig. II.

4. A Plane is perpendicular to another, when the Line perpendicular to the common section of the Planes, and drawn in the one, is also perpendicular to the other Plane.

We call the common Section of the Planes, a Line which is in both Planes; as the Line AB, which is also in the Plane AC, as well as in the Plane AD. If then the Line DE, drawn in the Plane AD, and perpendicular to AB, is also perpendicular to the Plane AC; the Plane AD shall be Right or perpendicular to the Plane AC. Fig. III.

5. If the Line AB be not perpendicular to the Plane CD, and if there be drawn from the Point A, the Line AE perpendicular to the Plane CD, and then the Line BE, the Angle ABE, is that of the inclination of the Line AB, to the Plane CD. Fig. IV.

6. The inclination of one Plane to another, is the Acute Angle comprehended by the two perpendiculars to the Fig. V.

common

common ſection drawn in each Plane. *As the inclination of the Plane AB, to the Plane AD, is no other than the Angle BCD, comprehended by the Lines BC CD, drawn in both Planes perpendicular to the common Section AE.*

7. Plains ſhall be inclined after the ſame manner, if the Angles of inclination are equal.

8. Planes which are parallel being continued as far as you pleaſe, are notwithſtanding equidiſtant.

9. Like ſolid figures, are comprehended, or terminated under like Planes equal in number.

10. Solid figures, equal and like, are comprehended or termined under like Plains equal both in Multitude and Magnitude.

Fig. VI. 11. A ſolid Angle is the concourſe or inclination of many Lines, which are in divers Planes. *As the concourſe of the Lines AB, AC, AD, which are in divers Planes.*

Fig. VI. 12. A Pyramid is a ſolid figure comprehended under divers Planes, ſet upon one Plane, and gathered together to one point. *As the figure ABCD.*

Fig. VII. 13. A Priſm is a ſolid figure contained under Planes, whereof the two oppoſite are equal, like, and parallel, but the others

others are parallelograms. *As the figure A B its oppoſite Plains may be Polygons.*

14. A Sphere is a ſolid figure terminated by a ſingle Superficies, from which drawing ſeveral Lines to a point taken in the middle of the figure, they ſhall be all equal; *Some define a Sphere by the motion of a ſemi-circle turned about its Diameter, the Diameter remaining immoveable.*

15. The Axis of a Sphere is that unmoveable Line about which the ſemi-circle turneth.

16. The Center of the Sphere, is the ſame with that of the ſemi-circle which turneth.

17. The Diameter of a Sphere is any Line whatever, which paſſeth through the Center of the Sphere, and endeth in its Superficies.

18. A Cone is a figure made, when one Side of a Right Angled Triangle, *viz.* one of thoſe that contain the Right Angle remaining fixed, the Triangle is turned round about till it return to the place from whence it firſt moved. And if the fixed Right Line be equal to the other which containeth the Right Angle, then the Cone is a Rectangled Cone, but if it be leſs, it is an Obtuſe Angled Cone;

Cone; if greater an Acute Angled Cone.

19. The Axis of a Cone is that fixed Line about which the Triangle is moved.

20. A Cylinder is a figure made by the moving round of a Right Angled Parallelogram, one of the sides thereof, namely which contains the Right Angle, abiding fixed; till the parallelogram be turned about to the same place whence it began to move.

21. Like Cones and Cylinders, are those whose Axes and Diameters of their Bases are Proportional. Cones are right, when the Axis is perpendicular to the Plain of the Base; and they are said to be Scalene, when the Axis is inclined to the Base, and the Diameter of their Bases are in the same Ratio: We add, that inclined Cones to be like their Axes, must have the same inclination to the Planes of their Bases.

PRO-

PROPOSITION I.

THEOREM.

A *Strait Line cannot have one of its parts in a Plane, and the other without it.* Plate VII. Prop. I.

If the Line A B be in the Plane A D, it being continued, ſhall not go without, but all its parts ſhall be in the ſame Plane. For if it could be that B C were a part of A B continued. Draw in the Plane C D, the Line B D, perpendicular to A B: draw alſo in the ſame Plane, B E perpendicular to B D.

Demonſtration. The Angles A B D, B D E, are both Right Angles; thence (*by the* 14*th. of the firſt*,) A B, B E, do make but one Line; and conſequently B C, is not a part of the Line A B continued; otherwiſe two ſtrait Lines C B, E B, would have the ſame part A B: that is A B would be part of both: which we have rejected as falſe in the Thirteenth Maxim of the firſt Book.

USE.

WE *eſtabliſh on this Propoſition a principle in Gnomonicks, to prove*

prove that the ſhadow of the ſtile falleth not without the Plane of a great Circle, in which the Sun is. Seeing that the end or top of the ſtile is taken for the Center of the Heavens; and conſequently for the Center of all the great Circles: the ſhadow being always in a ſtreight Line, with the Ray drawn from the Sun to the Opaque Body; this Ray being in any great Circle, the ſhadow muſt alſo be therein.

PROPOSITION II.

THEOREM.

LInes which cut one another, are in the ſame Plane, as well as all the parts of a Triangle.

If the Two Lines B E, C D, cut one another in the Point A; and if there be made a Triangle by drawing the Baſe B C: I ſay that all the parts of the Triangle A B C, are in the ſame plane, and that the Lines B E, C D, are likewiſe therein.

Demonſtration. It cannot be ſaid that any one part of the Triangle A B C, is in a Plane, and that the other part is without; without ſaying that one part of a Line

a Line is in one Plane, and that the other part of the ſame Line is not therein; which is contrary to the firſt Propoſition: and ſeeing that the ſides of the Triangle are in the ſame Plane wherein the Triangle is; the Lines B E, C D, ſhall be in the ſame Plane.

USE.

THis Propoſition doth ſufficiently determine a Plane, by two ſtreight Lines mutually interſecting each other, or by a Triangle; I have made uſe thereof in Opticks, to prove that the objective parallel Lines which fall on the Tablet, ought to be Repreſented by Lines which concur in a Point.

PROPOSITION III.

THEOREM.

THe common ſection of two Planes is a ſtreight Line.

If Two Planes A B, C D, cut one another, their common ſection E F ſhall be a ſtreight Line. For if it were not, take Two Points common to both Planes which let be E and F; and draw a ſtrait

ſtrait Line from the point E to the point F, in the Plane A B, which let be E H F. Draw alſo in the Plane C D a ſtreight Line from E to F; if it be not the ſame with the former, let it be E G F.

Demonſtration. Thoſe Lines drawn in the Two Planes are two different Lines, and they comprehend a ſpace; which is contrary to the Twelfth Maxim. Thence they are but one Line, which being in both Planes, ſhall be their common ſection.

USE.

This Propoſition is fundamental. We do ſuppoſe it in Gnomonicks, when we repreſent in a Dial, the Circles of the hours, marking only the common ſection of their Planes, and that of the Wall.

PROPOSITION IV.

THEOREM.

IF a Line be perpendicular to two other Lines which cut one another, it ſhall be alſo perpendicular to the Plane of thoſe Lines.

If the Line A B be perpendicular to the

the Lines C D, E F, which cut one another in the point B; in ſuch manner that the Angles A B C, A B D, A B E, A B F, be right, which a flat figure cannot repreſent; it ſhall be perpendicular to the Plane C D, E F; that is to ſay, that it ſhall be Perpendicular to all the Lines that are drawn in that Plane through the point B: as to the Line G B H. Let equal Lines be cut B C, B D, B E, B F; and let be drawn the Lines E C, D F, A C, A D, A E, A F, A G, and A H.

Demonſtration. The four Triangles A B C, A B D, A B E, A B F, have their Angles Right in the Point B; and the Sides B C, B D, B E, B F, equal with the ſide A B common to them all. Therefore their Baſes A C, A D, A E, A F, are equal (*by the 4th. of the 1ſt.*)

2. The Triangles E B C, D B F, ſhall be equal in every reſpect, having the Sides B C, B D, B E, B F, equal; and the Angles C B E, D B F, oppoſite at the vertex being equal; ſo then the Angles B C E, B D F, B F C, B F D, ſhall be equal (*by the 4th. of the firſt,*) and their Baſes E C, D F, equal.

3. The Triangles G B C, D B H, having their oppoſite Angles C B G, D B H, equal, as alſo the Angles B D H,

BCG, and the ſides BC, BD, they ſhall then have (*by the 26th. of the 1ſt.*) their Sides BG, BH, CG, DH, equal.

4. The Triangles ACE, AFD, having their ſides AC, AD, AE, AF, equal, and the Baſes EC, DF, equal, they ſhall have (*by the 8th. of the 1ſt.*) the Angles ADF, ACE, equal.

5. The Triangles ACG, ADH, have the Sides AC, AD, CG, DH, equal; with the Angles ADH, AGC: Thence they ſhall have their Baſes AG, AH, equal.

Laſtly, the Triangles ABH, ABG, have all their ſides equal; thence (*by the 27th. of the 1ſt.*) the Angles ABG, ABH, ſhall be equal, and the Line AB perpendicular to GH. So then the Line AB ſhall be perpendicular to any Line which may be drawn through the point B, in the Plane of the Lines CD, EF, which I call perpendicular to the Plane.

USE.

THis Propoſition cometh often in uſe in the firſt Book of Theodoſius; *for example, to Demonſtrate that the Axis of the World*

is perpendicular to the Plane of the Equinoctial. In like manner in Gnomonicks, we Demonſtrate by this Propoſition, that the Equinoctial is perpendicular to the Meridian in Horizontal Dials, it is not leſs uſeful in other Treatiſes; as in that of Aſtrolabes, or in the ſections of Stone.

PROPOSITION V.

THEOREM.

IF a Line be Perpendicular to three other Lines which cut one the other in the ſame Point, theſe three Lines ſhall be in the ſame Plane.

If the Line AB be perpendicular to three Lines BC, BD, BE, which cut one another in the point B, the Lines BC, BD, BE, are in the ſame plane. Let the plane AE be the Plane of the Lines AB, BE; and let CF be the Plane of the Lines BC, BD. If BE was the common ſection of the Two Planes, BE would be in the Plane of the Lines BC, BD, as we pretend it ſhould. Now if BE be not the common ſection; let it be BG.

Demonſtration. A B is perpendicular to the Lines BC, BD; it is then perpendicular to the Plane CF, (*by the 4th. and 5th. Def.*) A B ſhall be alſo perpendicular to BG. Now it is ſuppoſed that it is perpendicular to B E; thence the Angles A BE, ABG, would be right, and equal, and notwithſtanding the one is a part of the other. So then the Two Planes cannot have any other common ſection beſides B E; it is therefore in the Plane C F.

PROPOSITION VI.

THEOREM.

THE Lines which are perpendicular to the ſame Plane, are parallel.

If the Lines A B, C D, be perpendicular to the ſame plane E F; they ſhall be parallel. It is evident that the internal Angles A B D, B D C, are right; but that is not ſufficient, for we muſt alſo prove that the Lines A B, C D, are in the ſame Plane. Draw D G, perpendicular to B D, and equal to A B: Draw alſo the Lines B G, A G, A D.

Demonſtration.

Demonſtration. The Triangles ABD, BDG, have the ſides AB, DG, equal: BD is common; the Angles ABD, BDG, are right. Thence the Baſes AD, BG, are equal (*by the 4th. of the 1ſt.*) Moreover the Triangles ABG, ADG, have all their ſides equal; thence the Angles ABG, ADG, are equal; and ABG being right, ſeeing AB is perpendicular to the Plane, the Angle ADG is right. Therefore the Line DG is perpendicular to the three Lines CD, DA, DB, which by conſequence are in the ſame Plane (*by the 5th.*) Now the Line AB is alſo in the plane of the Lines AD, DC, (*by the ſecond*;) thence AB, CD, are in the ſame plane.

Coroll. Two Lines which are parallel are in the ſame Plane.

USE.

WE demonſtrate by this Propoſition, that the hour Lines are parallels amongſt themſelves, in all Planes which are parallel to the Axis of the World; as in the Polar and Meridian Dials, and others.

PROPOSITION VII.

THEOREM.

A Line which is drawn from one parallel to another, is in the ſame Plane which thoſe Lines are.

The Line C B being drawn from the point B of the Line A B, to the point C of its parallel C D: I ſay that the Line C B is in the Plane of the Lines A B, C D.

Demonſtration. The parallels A B, C D, are in the ſame Plane; in which if you draw a ſtrait Line from the point C to the point B, it will be the ſame with C B; otherwiſe two ſtrait Lines would incloſe a ſpace, *contrary to the* 12*th. Maxim.*

PROPOSITION VIII.

THEOREM.

IF of Two parallel Lines, the one be perpendicular to a Plane, the other ſhall be ſo likewiſe.

As if of the Two parallel Lines A B, C D; the one A B be perpendicular to the

the Plane EF: CD ſhall be ſo likewiſe. Draw the Line BD, ſeeing that ABD is a right Angle, and that the Lines AB, CD are ſuppoſed parallel; the Angle CDB ſhall be right (*by the* 30*th. of the* 1*ſt.*) Wherefore if I Demonſtrate that the Angle CDG is Right, I ſhall prove (*by the* 4*th.*) that CD is perpendicular to the Plane EF. Make the right Angle BDG, and make DG equal to AB, then draw the Lines PG, AG.

Demonſtration. The Triangles ABD, BDG, have their ſides AB, DG, equal; the ſide BD is common, the Angles ABD, BDG, are right; Thence (*by the* 4*th. of the* 1*ſt.*) the Baſes AD, BG, are equal. The Triangles ADG, ABG, have all their ſides equal; ſo then (*by the* 6*th.*) the Angles ADG, ABG, are equal. This laſt is right, ſeeing the Line AB is ſuppoſed perpendicular to the Plane EF; thence the Angle ADG, is right; and the Line DG being perpendicular to the Lines DB, DA, it ſhall be perpendicular to the Plane of the Lines AD, DB; which is the ſame in which are the parallels AB, CD. So then the Angle GDC is a Right Angle (*by the* 5*th. Def.*)

PRO-

PROPOSITION IX.

THEOREM.

THe Lines which are Parallel to a third, are parallel amongſt themſelves.

If the Lines A B, C D, are parallel to the Line E F; they ſhall be parallel to each other, although they be not all three in the ſame Plane. Draw in the Plane of the Lines A B, E F, the Line H G perpendicular to A B, it ſhall be alſo perpendicular to E F, (*by the* 29*th. of the firſt.*) In like manner draw in the Plane of the Lines E F, C D, the Line H I perpendicular to E F, C D.

Demonſtration. The Line E H being perpendicular to the Lines H G, H I; is alſo perpendicular to the Planes of the Lines H G, H I; (*by the* 4*th.*) thence (*by the* 8*th.*) the Lines A G, C I, are ſo likewiſe, and (*by the* 6*th.*) they ſhall be parallel.

USE.

THis Propoſition ſerveth very often in Perſpective, to determine on the Plane

Plane the Image of the parallel Lines, and in the ſections of ſtones, where we prove that the ſides of the Squares are parallel to each other, becauſe they are ſo to ſome Line in a different Plane. In Gnomonicks we prove that vertical Circles ought to be drawn by perpendiculars on the Wall, becauſe that the Lines which are their common ſection with the Wall, are parallel to the Line drawn from the Zenith to the Nadir.

PROPOSITION X.

THEOREM.

IF Two Lines which meet in a point are parallel to Two other Lines in a different Plane; they will form an equal Angle.

If the Lines AB, CD; AE, CF, are parallel, although they be not all four in the ſame Plane; the Angles BAE, DCF, ſhall be equal. Let the Lines AB, CD; AE, CF, be equal: and draw the Lines BE, DF, AC, BD, EF.

Demonſtration. The Lines AB, CD, are ſuppoſed parallel and equal; thence (*by the 33d. of the firſt,*) the Lines AC, BD,

BD, are parallel and equal; as alſo AC, and EF; and (*by the preceding*) BD, EF, ſhall be alſo parallel and equal; and (*by the 33d. of the firſt,*) BE, DF, ſhall be parallel and equal. So then the Triangles BAE, DCF, have all their ſides equal; and (*by the 8th.*) the Angles BAE, DCF, ſhall be equal.

Coroll. One might make ſome Propoſitions like unto this, which would not be unuſeful; as this. If one ſhould draw in a parallel Plane, the Line CD parallel to AB, and if the Angles BAC, DCF, are equal; the Lines AC, CF, are parallel.

USE.

WE Demonſtrate by this Propoſition, that the Angles which are made by the hour Circles in a Plane parallel to the Æquator, are equal to thoſe which they make in the Plane of the Æquator.

PRO-

PROPOSITION XI.

PROBLEM.

TO draw a perpendicular to a Plane, from a point given without the Plane.

If you would from the point C, draw a perpendicular to the Plane AB; draw the Line EF at diſcretion, and CF perpendicular thereto (*by the 12th. of the firſt.*) then draw (*by the 11th. of the firſt,*) in the Plane AB, the Line FG perpendicular to ED, and CG perpendicular to FG. I demonſtrate that CG ſhall be perpendicular to the Plane AB. Draw GH parallel to EF.

Demonſtration. The Line EF being perpendicular to the Lines CF, FG, it ſhall be perpendicular to the Plane CFG (*by the 4th.*) and HG being parallel to EF, ſhall be alſo perpendicular to the ſame Plane (*by the 8th.*) and ſeeing that CG is perpendicular to the Lines GF, GH, it ſhall be perpendicular to the Plane AB, (*by the 4th.*)

PRO

PROPOSITION XII.

PROBLEM.

TO draw a perpendicular to a Plane, from a point of the same Plane.

To draw from the point C a perpendicular to the Plane AB; draw from the point E taken at discretion without the Plane, ED perpendicular to the same Plane (*by the* 11*th.*) Draw also (*by the* 30*th. of the first,*) CF parallel to DE, CF shall be perpendicular to the Plane AB, (*by the* 8*th.*)

PROPOSITION XIII.

THEOREM.

THere cannot be drawn from the same point Two Perpendiculars to a Plane.

If the Two Lines CD, CE, drawn from the same point C, were perpendicular to the Plain AB; and that CE was the common section of the Plane of those Lines, with the Plane AB; the Angles ECF,

ECF, DCF, would be Right; which is impoſſible.

I further add, that there cannot be drawn from the ſame point D Two perpendiculars DC, DF, to the ſame Plane AB: for having drawn the Line CF, there will be made Two Right Angles DCF, DFC, in one Triangle contrary to the 32*d.* of the firſt.

USE.

THis Propoſition is neceſſary to ſhew, that a Line which is drawn perpendicular to a Plane is well determined, ſeeing there can be drawn but only one from a Point.

PROPOSITION XIV.

THEOREM.

PLanes are parallel to which the ſame Line is perpendicular.

If the Line AB be perpendicular to the Planes AC, BD, they ſhall be parallel; that is to ſay, they ſhall be in all places equidiſtant. Draw the Line DC parallel

parallel to A B, (*by the 30th. of the first.*) and joyn the Lines B D, A C.

Demonstration. It is supposed that A B is perpendicular to the Planes A C, B D; thence the Line C D, which is parallel thereto, shall be also perpendicular to them (*by the 8th.*;) so then the Angles B and D, A and C; shall be Right; and (*by the 29th. of the first.*) the Lines A C, B D, shall be parallel. The figure A B C D shall be thence a parallelogram. Now (*by the 34th. of the first.*) the Lines A B, C D, are equal; that is to say, that the Planes in the Points A and B, C and D, are equi-distant: So then because we can draw the Line C D from any point whatever, the Planes A C, B D, shall be equi-distant in all places.

USE.

THeodosius *Demonstrateth by this, that the Circles which have the same Poles, as the Equinoctial, and the Tropicks, are parallel; because the Axis of the World is perpendicular to their Planes.*

PRO-

PROPOSITION XV.

THEOREM.

IF Two Lines which meet in the ſame point are parallel to Two Lines of another plane; the Planes of thoſe Lines ſhall be parallel.

If the Lines AB, AC, in the one Plane, are parallel to the Lines DF, DE, which are in another Plane; the Planes BC, FE, are parallel. Draw AI perpendicular to the Plane BC (*by the* 11*th.*) and GI, IH, parallel to FD, DE; they ſhall alſo be parallel to the Lines AB, AC, (*by the* 9*th.*)

Demonſtration. The Lines AB, GI, are parallel, and the Angle IAB is Right, ſeeing AI is perpendicular to the Plane BC: thence (*by the* 29*th. of the firſt*) the Angle AIG is Right. AIH is alſo Right: Thence (*by the* 4*th.*) the Line AI is perpendicular to the Plane GH; and it being alſo perpendicular to the Plane BC, the Planes BC, FE, ſhall be parallel (*by the* 14*th.*)

PRO-

PROPOSITION XVI.

THEOREM.

IF a Plane cut Two Planes which are parallel, its common Sections with them shall be parallel Lines.

If the Plane AB cutteth two other parallel Planes AC, BD: I demonstrate that the common sections AF, BE, shall be parallels. For if they were not, they would meet if continued, for example in the point G.

Demonstration. The Lines AF, BE, are in the Planes AC, BD, and do not go out of the same (*by the first*;) wherefore if they meet each other in G, the Planes would meet each other likewise, and consequently they would not be parallel, contrary to what we have supposed.

USE.

WE Demonstrate by this Proposition, in the Treatise of Conick Sections and Cylindricks, that the Cone or Cylinder being cut by a parallel to its Base, the Sections are Circles; we describe Astrolabes:

We

We prove in Gnomonicks, that the Angles the hour Circles make with a Plane parallel to a great Circle, are equal to those which they make with the Circle it self; we demonstrate in Perspective, that the Image of the objective Lines which are perpendicular to the Tablet, concur in the point of vision.

PROPOSITION XVII.

THEOREM.

TWo Lines are divided proportionally by parallel Planes.

Let the Lines A B, C D, be divided by parallel Planes; I say that A E, E B, and C F, F D, are in the same Ratio. Draw the Line A D, which meeteth the Plane E F in the point G: draw also A C, B D, F G, E G.

Demonstration. The Plane of the Triangle A B D, cutteth the three Planes; thence *(by the 16th.)* the sections B D, E G, shall be parallel; and *(by the 2d. of the sixth,)* there shall be the same Ratio of A E to E B, as of A G to G D. In like manner, the Triangle A D C cutteth the Planes E F, A C; thence the sections A C, G F, are parallel; and there shall be the same Ratio of F C to F D, as of A G

AG to GD, that is to say, as of AE to EB.

PROPOSITION XVIII.

THEOREM.

IF a Line be perpendicular to a Plane, all the Planes in which that Line shall be found, shall be perpendicular to the same Plane.

If the Line AB be perpendicular to the Plane ED; all the Plains in which it shall be found, shall be also perpendicular to ED. Let AB be in the Plane AE, which hath for common section with the Plane ED, the Line BE, unto which let there be drawn the perpendicular FI.

Demonstration. The Angles ABI, BIF, are Right; thence the Lines AB, FI, shall be parallel; and (*by the 8th.*) FI shall be perpendicular to the Plane ED. So then the Plane AE shall be perpendicular to the Plane ED, (*by the 5th. Def.*)

USE

USE

THE firſt Propoſition in Gnomonicks, and which may ſtand for a fundamental. is eſtabliſhed on this Propoſition, it is alſo often made uſe of in Spherical Trigonometry, in perſpective and generally in all Treatiſes, which are obliged to conſider ſeveral Planes.

PROPOSITION XIX.

THEOREM.

IF Two Planes which cut each other, are perpendicular to another, their common ſection ſhall alſo be perpendicular thereto.

If the Two Planes A B, E D, cut each other, and are perpendicular to the Plane I K; there common ſection E F is perpendicular to the Plane I K.

Demonſtration. If E F be not perpendicular to the Plane I K; let there be drawn in the Plane A B, the Line G F perpendicular to the common ſection B F; and ſeeing the Plane A B is perpendicular to the Plane I K; the Line G F, ſhall be perpendicular to the ſame Plane.

Let

Let there alſo be drawn F H perpendicular to the common ſection D F, it ſhall be perpendicular to the Plane I K. Thus we ſhall have Two perpendiculars to the ſame Plane, drawn from the ſame point F, *contrary to the* 13*th. Propoſition.* We muſt therefore conclude that E F is perpendicular to the Plane I K.

USE.

WE demonſtrate by this Propoſition, that the Circle which paſſeth through the Poles of the World, and through the Zenith, is the Meridian, and cutteth in two equally all the Diurnal Arks, and that every ſtar employes the ſame ſpace of time from their riſing to this Circle, as they do from this Circle to their ſetting.

PROPOSITION XX.

THEOREM.

IF three Plain Angles compoſe a ſolid Angle, any Two of them ſhall be greater than the third.

If the Angles B A C, B A D, C A D, Compoſe the ſolid Angle A; and if B A C be the greateſt of them; the other Two B A D,

BAE, CAD, taken together, are greater than BAC. Let CAE be equal to CAD, and let the Lines AD, AE, be equal. Draw the Lines CEB, CD, BD.

Demonſtration. The Triangles CAE, CAD, have the ſides AD, AE, equal; CA, common; and the Angles CAD, CAE, equal; thence (*by the 4th. of the firſt,*) the Baſes CD, CE, are equal. Now the Sides CD, BD, are greater than the ſide BC (*by the 20th of the firſt,*) wherefore taking away the equal Lines CD, CE; the Line BD ſhall be greater than BE. Moreover the Triangles BAE, BAD, have their ſides AD, AE, equal; AB common; and the Baſe BD greater than EB: thence (*by the 18th. of the firſt*;) the Angle BAD is greater than the Angle BAE; and adding the Angles CAD, CAE; the Angles BAD, CAD, ſhall together be greater than BAE, CAE, that is to ſay, than CAB.

PRO-

PROPOSITION XXI.

PROBLEM.

ALL the Plain Angles which Compose a Solid Angle are less than four Right.

If the Plane Angles B A C, C A D, B A D, Compose the solid Angle A; they shall be less than four Right. Draw the Lines B C, C D, B D, and you shall have a Pyramid, which hath for Base the Triangle B C D.

Demonstration. There is made a solid Angle at the point B, of which the Angles A B C, A B D, taken together, are greater than the Angle C B D at the Base. In like manner A C B, A C D, are greater than B C D; and the Angles A D C, A D B, are greater than C D B. Now all the Angles at the Base C D B, are equal to Two Right; thence the Angles A B C, A B D, A C B, A C D, A D C, A D B, are greater than Two Right. And because all the Angles of the Three Triangles B A C, B A D, C A D, are equal to six Right; in taking away more than Two Right, there

there will remain less than four Right for the Angles which are made at the point A. If the solid Angle A was Composed of more than Three plain Angles; in such manner that the Base of the Pyramid was a Polygon; it might be divided into Triangles: and by making the supputation, one would always find that all the Plane Angles which form the solid Angle, taken together, are less than four Right.

USE.

These Two Propositions serve to determine, when of several plain Angles there may be made a solid Angle, which is often necessary in the Treatise of cutting of stone, and in the following Propositions.

The Two and Twentieth and Three and Twentieth Propositions are needless.

PROPOSITION XXIV.

THEOREM.

If a solid Body be termined by parallel Planes, the opposite Planes shall be like Parallelograms, and equal.

If the solid A B be terminated by parallel

rallel Planes, the oppoſite Superficies ſhall be like and equal parallelograms.

Demonſtration. The parallel Planes AC, BE, are cut by the Plane FE; wherefore the common ſections AF, DE, are parallel (*by the* 15*th.*) In like manner DF, AE, whence AD ſhall be a parallelogram. I demonſtrate after the ſame manner, that AG, FB, CG, and the reſt, are parallelograms. I further add, that the oppoſite parallelograms; for example AG, BF, are like and equal. The Lines AE, EG, are parallel to the Lines FD, BD, and alſo equal; thence the Angles AEG, FDB, are equal (*by the* 15*th.*) I may alſo demonſtrate that all the ſides, and all the Angles of the oppoſite parallelograms are equal. Thence the Parallelograms are like and equal.

PROPOSITION XXV.

THEOREM.

IF a parallelepipedon be divided by a Plane parallel to one of its ſides; the Two ſolids which reſult from this Diviſion, ſhall be in the ſame Ratio as their Baſes.

If the parallelepipedon AB be divi-

ded

ded by the Plane C D parallel to the oppoſite Planes A F, B E, the ſolid A C to B D, ſhall be in the ſame Ratio as the Baſe A I to the Baſe D G. Let us imagine that the Line A H, which repreſents the height of the figure is divided into as many parts as one pleaſeth; for example in Ten Thouſand, which we may take to be indiviſible, that is to ſay, without thinking that they can be divided. Let us alſo ſuppoſe or imagine as many Superficies parallel to the Baſe AG, as there is parts in the Line A H: I expreſs but one for all, as O S; in which manner the Solid A B may be Compoſed of all thoſe Surfaces of equal thickneſs, as would be a Ream of Paper compoſed of all its Sheets, when laid on each other. It is evident that then the Solid A C ſhall be compoſed of Ten Thouſand Surfaces equal to the Baſe A I, (*by the preceeding*;) and the Solid B D ſhall contain Ten Thouſand Superficies equal to the Baſe D G.

Demonſtration. Each Surface of the Solid A C, hath the ſame Ratio to each Surface of the Solid B D, as the Baſe A I hath to the Baſe D G; ſeeing each of them are equal to their Baſes; thence (*by the 12th. of the 5th.*) all the Superficies

ficies of the Solid A C taken together, ſhall have the ſame Ratio to all the Surfaces of the Solid D B, as hath the Baſe A I to the Baſe D G. Now all the Surfaces of the Solid A C, compoſe the Solid A C, which hath no other parts but its Surfaces, and all the Surfaces of the Solid D B, are nothing elſe but the Solid D B; thence the Solid A C to the Solid D B, hath the ſame Ratio as hath the Baſe A I to the Baſe D G.

USE

THis way of Demonſtration is that of Cavallerius: *I find it to be very clear, provided it be made uſe of as it ought, and that the Line which is the meaſure of the thickneſs of its ſurfaces, be taken in the ſame manner in the one and in the other Term. I will make uſe thereof hereafter, to make more eaſie ſome Demonſtrations which otherwiſe are too incombred.*

PROPOSITION XXVI.

THEOREM.

A Parallelepipedon is divided into Two equally, by a Diagonal Plane.

Let the Parallelepipedon A B be divided

ded by the Plane C D, drawn from one Angle to the other: I ſay that it divideth the ſame into Two equally. Let the Line EA, be ſuppoſed to be divided into as many parts as one liſteth; and that from each, one hath drawn as many Planes parallel to the Baſe A F; each of thoſe Planes is a Parallelogram equal to the Baſe A F (*by the* 24*th.*)

Demonſtration. All the Parallelograms which can be drawn parallel to the Baſe A F, are divided into Two equally by the Plane C D; for the Triangle which will be made on each ſide of the Plane C D, have their common Baſe equal to C G; and their ſides equal, ſeeing they are thoſe of a parallelogram. Now it is evident that the Parallelepipedon A B, containeth nothing elſe but its parallelogram Surfaces, each of which is divided into Two equal Triangles: therefore the parallelepipedon A B, is divided into Two equally by the Plane C D.

The Twenty Seventh and Twenty Eighth Propopoſitions are uſeleſs, according to this way of Demonſtration.

PROPOSIT. XXIX. XXX. & XXXI.

THEOREM.

Parallelepipedons having the ſame height and the ſame, or equal Baſes, are equal.

If the Parallelepipedon CD, AB, have equal perpendicular height AE, FG, with equal Baſes AH, CI, or the ſame, they ſhall be equal. Let be put the Two Baſes AH, CI, on the ſame Plane, ſeeing their heights are equal, the Baſes EB, FD, ſhall be in the ſame Plane, parallel to that of the Baſes AH, CI. Let it be imagined that the perpendicular FG or EA is divided into as many equal parts as one liſteth; for example in Ten Thouſand; and there be drawn through each, Surfaces or Planes of the ſame thickneſs; to expreſs which I mention but one for all, which ſhall be KM. Each Surface ſhall form in each Solid a Plane parallel like and equal its Baſe (*by the 24th*) as KL, OM, and there ſhall be as many of them in the one as in the other; ſeeing their thickneſs which I take perpendicularly in the Line of their height, is equal.

Demonſtration.

Demonſtration. There is the ſame Ratio of the Baſe A H to the Baſe C I, as of each plain K L to O M: Now there is the ſame number of them in the one as there is in the other, ſo then (*by the 15th. of the fifth*) there ſhall be the ſame Ratio of all the Antecedents to all the Conſequents, that is to ſay of all the Solid A B to all the Solid C D; as of the Baſe A H to the Baſe C I. Now it is ſuppoſed that the Baſes are equal: thence the Solids A B, C D, are equal.

Coroll. To find the Solidity of a Parallelepipedon, we Multiply the Baſe by its height taken perpendicularly, becauſe that perpendicular ſheweth how many times their is found Surfaces equal to the Baſe. As if I take a Foot for an indiviſible meaſure, that is to ſay, which I will not conceive ſubdivided: If the Baſe ſhould contain Twelve Foot Square, and that the perpendicular height were Ten Foot; I ſhould have One Hundred and Twenty Cubick Feet, for the Solidity of the Body A B. For ſeeing the height A E, is Ten Foot; I may make Ten, parallelograms equal to the Baſe, and which ſhall have each of them one Foot thickneſs. Now the Baſe with one Foot thickneſs, makes Twelve

Foot Cubick; it ſhall then contain One Hundred and Twenty, if it hath Ten Foot height.

PROPOSITION XXXII.

THEOREM.

Parallelepipedons of the ſame height are in the ſame Ratio as their Baſes.

I have Demonſtrated this Propoſition in the precedent, in proving that there is the ſame Ratio of the Parallepipedon AB to the Parallelepipedon CD, as of the Baſe AH to the Baſe CI.

Coroll. Parallelepipedons which have equal Baſes, are in the ſame Ratio as are their height; as the Parallelepipedons AB, AL, which have for their Perpendicular height AK, AE; for if the height AK were divided into as many Aliquot parts as one would, and AE into as many equal parts of the firſt, as it would contain, and if there were drawn through each as many Planes parallel to the Baſe, I ſay as many as AE would contain of Aliquot parts of AK, ſo many Solids AB ſhall contain of Surfaces equal to the Baſe, which are Aliquot

parts

parts of the Solid AL. Therefore *(by the 5th. of the 5th.)* there ſhall be the ſame Ratio of the Solid AB to the Solid AL, as of the height to the height AK.

USE

THe Three precedent Propoſitions contain almoſt all the menſuration of Parallelepipedons, and ſerve as the firſt principles in this matter. It is thus we meaſure the Solidity of a Wall, by Multiplying their Baſes by their heights.

PROPOSITION XXXIII.

THEOREM.

LIke Parallelepipedons are in tripled Proportion of their Homologus ſides.

If the Parallelepipedons AB, CD, are like; that is to ſay, if all the Planes of the one are like unto all the Planes of the other; and if all their Angles are equal, in ſuch manner that they may be placed in a ſtrait Line, that is to ſay, that AE, EF; HE, EI; GE, EC; be ſtrait Lines, and that there is the ſame Ratio of AE to EF, as of HE to EI,

and of G E to E C. I demonſtrate that four Solids are continually proportionals, according to the Ratio of E A, to that which is homologous thereto, which is E F or D I.

Demonſtration. The Parallelepipedon A B, hatth the ſame Ratio to E L of the ſame height, as hath the Baſe A H to the Baſe E O (*by the* 32*d.*) Now the Baſe A H to the Baſe E O, hath the ſame Ratio as A E to E F, (*by the* 1*ſt. of the* 6*th.*) In like manner the Solid E L to the Solid E K, is the ſame as that of the Baſe E O to the Baſe E D, that is to ſay, as of H E to E I. In fine the Solid E K to the Solid E N, hath the ſame Ratio, as hath the height G E to the height E C, (*by the coroll.*) or (taking E F for their common height,) as of the Baſe G I to the Baſe C I, that is to ſay, as of G E to E C. Now the Ratio of A E to E F, and of H E to E I, and of G E to E C, is the ſame, as we did ſuppoſe: By conſequence, there is the ſame Ratio of A B to E L, as of E L to E K, and of E K to C D. Therefore (*by the* 11*th. Def. of the* 5*th.*) the Ratio of A B to C D, ſhall be triple of that of A B to E L, or of A E to his homologous E F.

Coroll.

Coroll. If four lines are continually proportional, the Parallelepipedon described on the first, hath the same Ratio to a like Parallelepipedon described on the second, as the first hath to the fourth, for the Ratio of the first to the fourth is tripled of that of the first to the second.

USE.

YOu may comprehend by this Proposition, that the Celebrated Proposition of the Duplication of the Cube, proposed by the Oracle, consisteth in this to find two means continually Proportional. For if you propose for this first term, the side of the first Cube; and that the fourth term be the double of this first: If you find Two mean proportionals, the Cube described on the first Line shall have the same Ratio to that described on the second, as the first Line hath to the fourth, which shall be as One to Two. We also correct by this Proposition the false opinion of those which imagine, that like Solids are in the same Ratio as are their sides, as if a Cube of one foot long were the half of a Cube of Two Foot long, although it be but the eighth part thereof. This is the Principle of the Calibres, which

may be applyed not only to Cannon Shot, but alſo to all like Bodies. For example, I have ſeen a perſon which would make a Naval Architecture, and would keep the ſame Proportions in all ſorts of Veſſels: but argued thus; if a Ship of One Hundred Tuns, ought to be Fifty Foot long, a Ship of Two Hundred ought to be One Hundred Foot by the Keel. In which he was much miſtaken, for inſtead of making a Ship of twice that Burden, he would make one Octuple. He ought to have given to the Second Ship to be double in Burden to the Firſt, ſomewhat leſs than Sixty Three Foot.

PROPOSITION XXXIV.

THEOREM.

EQual Parallelepipedons have their Baſes and height Reciprocal, and thoſe which have their Baſes and height Reciprocal, are equal.

If the Parallelepipedons AB, CD, are equal, they ſhall have their Baſes and height reciprocal; that is to ſay, there ſhall be the ſame Ratio of the Baſe AE to the Baſe CF, as of the height CH to the height AG. Having made CI

CI equal to AG, draw the Plane IK parallel to the Baſe CF.

Demonſtration. The Parallelepipedon AB, hath the ſame Ratio to CK of the ſame height, as hath the Baſe AE to CF, (*by the* 32*d.*) Now as AB to CK, ſo is CD to the ſame CK, ſeeing that AB and CD are equal; and as CD is to CK, which hath the ſame Baſe, ſo is the height CH to the height CI (*by the Coroll. of the* 32*d.*) Wherefore, as the Baſe AE is to the Baſe CF, ſo is the height CH to the height CI or AG.

I further add, that if there be the ſame Ratio of AE to CF, as of the height CH to the height AG; the Solids AB, CD, ſhall be equal.

Demonſtration. There is the ſame Ratio of AB to CK, of the ſame height, as of the Baſe AE to the Baſe CF, (*by the* 32*d.*) there is alſo the ſame Ratio of the height CH to the height CI or AG, as of CD to CK; we ſuppoſe that the Ratio of AE to CF, is the ſame of that of CH to CI or AG; ſo then there ſhall be the ſame Ratio of the Solid AB to the Solid CK, as of the Solid CD to the ſame Solid CK. Therefore (*by the* 8*th. of the* 5*th.*) the Solids AB, CD, are equal.

USE.

THis reciprocation of Baſes, rendreth theſe Solids eaſie to be meaſured; it hath alſo a kind of Analogy with the Sixteenth Propoſition of the Sixth Book which beareth thus, that parallelograms equiangled and equal, have their Sides reciprocal, and it Demonſtrateth alſo as well the practice of the Rule of Three.

PROPOSITION XXXVI.

THEOREM.

IF Three Lines are continually proportional, the Solid Parallelepipedon made of them is equal to an equiangular parallelepipedon, which hath all it ſides equal to the middle Line.

If the Lines A, B, C, be continually proportional, the parallelepipedon F E made of them, that is to ſay, which hath the ſide F I equal to the Line A, and H E equal to B, H I equal to C; is equal to the equiangular Parallelepipedon K L, which hath its ſides L M, M N, K N,

equal to the Line B. Let there be drawn from the Points H and N, the Lines H P, N Q, perpendicular to the Planes of the Baſes; they ſhall be equal, ſeeing that the Solid Angles E and K are ſuppoſed equal, (in ſuch manner that if they did penetrate each other, they would not ſurpaſs each other) and that the Lines E H, K N, are ſuppoſed equal. Therefore the heights H P, N Q, are equal.

Demonſtration. There is the ſame Ratio of A to B, or of F I to L M, as of B to C, or of L M to H I; ſo then the parallelogram F H comprehended under F I, I H, is equal to the parallelogram L N, comprehended under L M, M N, equal to B (*by the* 15*th. of the* 6*th.*) the Baſes are thence equal. Now the heights H P, N Q, are ſo likewiſe; therefore (*by the* 31*ſt.*) the Parallelepipedons are equal.

PROPOSITION XXXVII.

THEOREM.

IF four Lines are proportional, the Parallelepipedons deſcribed on thoſe Lines ſhall be Proportional, and if like Parallelepipedons are Proportional, their homologous ſides ſhall be ſo likewiſe.

If A to B be in the ſame Ratio as C to D; the like Parallelepipedons which ſhall have for their homologous ſides A, B, C, D, ſhall be in the ſame Ratio.

Demonſtration. The Parallelepipedon A to the Parallelepipedon B, is in triple Ratio of that of the Line A to the Line B, or of that of the Line C to the Line D. Now the Parallelepipedon C to the Parallelepipedon D, is alſo in triple Ratio of that of C to D, (*by the* 33*d.*) there is therefore the ſame Ratio of the Parallelepipedon A to the Parallelepipedon B, as of the Parallelepipedon C to the Parallelepipedon D.

PROPOSITION XXXVIII.

THEOREM.

IF Two Planes be Perpendicular to each other; the Perpendicular drawn from one Point of one of the Planes to the other, ſhall fall on the common ſection.

If the Planes A B, C D, being perpendicular to each other, there be drawn from the Point E of the Plane A B, a Line perpendicular to the Plane C D; it ſhall fall on A G, the common ſection of

of the Planes. Draw E F perpendicular to the common ſection A G.

Demonſtration. The Line E F perpendicular to A G, the common ſection of the Planes, which we ſuppoſed perpendicular, ſhall be perpendicular to the Plane C D (*by the* 3*d. Def.*) and ſeeing there cannot be drawn from the Point E two perpendiculars to the Plane C D (*by the* 13*th.*) the perpendicular ſhall fall on the common Section A G.

USE.

THis Propoſition ought to have been after the 17*th. ſeeing it hath reſpect to Solids in general. It is of uſe in the Treatiſe of Aſtrolabes, to prove that in the Analemma, all the Circles which are perpendicular to the Meridian, are ſtrait Lines.*

PROPOSITION XXXIX.

THEOREM.

IF there be drawn in a parallelepipedon Two Planes dividing the oppoſite ſides equally; their common Section and the Diameter doth alſo cut each other equally.

Let

Let the oppoſite ſides of the parallelepipedon A B, be divided into Two equally by the Planes C D, E F; their common ſection G H, and the Diameter B A ſhall divide each other equally in O. Draw the Lines B G, G K, A H, L H. I prove in the firſt place, that they make but one ſtrait Line; for the Triangles D B G, K M G, have their ſides B D, K M, equal, ſeeing they are the halves of equal ſides; as alſo G D, G M. Moreover, D B, K M, being parallel, the Alternate Angles B D G, G M K, ſhall be equal (*by the* 29*th. of the firſt*;) ſo then (*by the* 4*th. of the firſt*,) the Triangles D B G, K G M, ſhall be equal in every reſpect; and conſequently the Angles B G D, K G M. Now (*by the* 16*th. of the* 1*ſt.*) B G, G K, is but one Line, as alſo L H, H A; thence A L, B K, is but one Plane, in which is found the Diameter A B and G H, the common ſection of the Planes. The Plane A L, B K, cutting the parallel Planes A N, C D, ſhall have its common ſections G H, A K, parallel; and (*by the* 4*th. of the* 6*th.*) there ſhall be the ſame Ratio of B K to G K, as of B A to B O; and of A K to O G, (*by the* 4*th. of the* 6*th.*) Now B K is double to B G;

BG; thence BA is double to BO; as AK equal to HG, is double to GO. So then the Lines GH, AB, cut each other equally in the Point O.

Coroll. 1. All the Diameters are divided in the Point O.

Coroll. 2. Here may be put *Corollaries* which depend on ſeveral Propoſitions; for example, that Triangular Priſms of equal height, are in the ſame Ratio as are their Baſes; for the Parallelepipedons of which they are the halves (*by the* 32*d.*) are in the ſame Ratio as their Baſes; ſo that the half Baſes and the half parallelepipedons, that is to ſay the Priſms, ſhall be in the ſame Ratio.

Coroll. 3. Polygon Priſms of the ſame height, are in the ſame Ratio as are their Baſes; ſeeing they may be reduced to Triangular Priſms, which ſhall each of them be in the ſame Ratio as their Baſes.

Coroll. 4. There may be applied to the Priſm the other Propoſitions of the Parallelepipedons; for example, that equal Priſms have their heights and Baſes Reciprocal, that like Priſms are in triple Ratio of their homologous ſides.

USE.

USE.

This Propoſition may ſerve to find the Center of gravity of Parallelepipedons, and to Demonſtrate ſome Propoſitions of the Thirteenth and Fourteenth Books of Euclid.

PROPOSITION XL.

THEOREM.

That Priſm which hath for its Baſe a Parallelogram double to the Triangular Baſe of another, and of the ſame height, is equal thereto.

Let there be propoſed Two Triangular Priſms ABE, CDG, of the ſame height; and let be taken for the Baſe of the firſt the Parallelogram AE, double to the Triangle FGC, the Baſe of the ſecond Priſm. I ſay that thoſe Priſms are equal. Imagine that the Parallelepipedons AH, GI, are drawn.

Demonſtration. It is ſuppoſed that the Baſe AE is double to the Triangle FGC: Now the Parallelogram GK being alſo double to the ſame Triangle (*by*

(*by the* 34*th. of the* 1*ſt.*) the Parallelograms A E, G K, are equal: and conſequently the Parallelepipedons A H, G I, which have their Baſes and heights equal, are equal; and Priſms which are the halves (*by the* 28*th.*) ſhall be alſo equal.

The End of the Eleventh Book.

THE

THE TWELFTH BOOK OF Euclid's *Elements*.

EUclid, *after having given in the preceding Books, the general Principles of Solids, and explained the way of measuring the most Regular, that is to say those which are terminated by plain Superficies; treateth in this, of Bodies incloſed under curved Surfaces, as are the Cylinder, the Cone, and the Sphere, and comparing them with each other, he giveth the Rules of their Solidity, and the way of meaſuring of them. This Book is very uſeful, ſeeing we find herein thoſe Principles whereon the moſt skilful Geometricians have eſtabliſhed ſo many curious Demonſtrations on the Cylinder, the Cone, and of the Sphere.*

PRO-

PROPOSITION I.

THEOREM.

Like Polygons inſcribed in a Circle, are in the ſame Ratio as are the Squares of the Diameters of thoſe Circles.

If the Polygons ABCDE, GFHKL, Plate VIII. inſcribed in Circles, are like; they ſhall be in the ſame Ratio as are the Squares of the Diameters A M, F N. Draw the Lines B M, G N, F H, A C.

Demonſtration. It is ſuppoſed that the Polygons are like, that is to ſay that the Angles B, and G, are equal; and that there is the ſame Ratio of A B to B C, as of F G to G H, whence I conclude (*by the 6th. of the 6th.*) that the Triangles A B C, F G H, are equiangular, and that the Angles A C B, F H G, are equal; ſo then (*by the 21ſt. of the 3d.*) the Angles A M B, F N G, are equal. Now the Angles A B M, F G N, being in a ſemi-circle, are Right (*by the 31ſt. of the 3d.*) and conſequently the Triangles A B M, F G N, are equiangular, Therefore (*by the 5th. of the 6th.*) there ſhall be the ſame Ratio of

A B

AB to FG, as of AM to FN; and (*by the 22d. of the 11th.*) if one deſcribe Two ſimilar Polygons on AB, FG, which are thoſe propoſed; and alſo Two other like Polygons AM, and FN, which ſhall be their Squares, there ſhall be the ſame Ratio of the Polygon ABCDE to the Polygon FGNKL, as of the Square of AM to the Square of FN.

This Propoſition is neceſſary to Demonſtrate the following Propoſition.

LEMMA.

IF a Quantity be leſs than a Circle, there may be inſcribed in the Circle a Regular Polygon, greater than that quantity.

Lemma. Fig. II. *Let the Figure A be leſs than the Circle B; there may be inſcribed in the Circle a Regular Polygon greater than the figure A. Let the figure G, be the difference of the figure A, and the Circle in ſuch ſort that the figures A and G taken together be equal to the Circle B. Inſcribe in the Circle B, the Squares C D E F* (by the 6th. of the 4th.)

4th.) *if that Square were greater than the figure A; we should have what we pretend to. If it be less, divide the Quadrants of the Circle C D, D E, E F, F C, into Two equally in the Points H, I, K, L, in such sort as you may have an Octogon. And if that Octogon be less than the figure A; subdivide the Arks, and you shall have a Polygon of Sixteen Sides, then Thirty Two of Sixty Four, &c. I say in fine you shall have a Polygon greater than the figure A; that is to say, a Polygon having less difference from the Circle than the figure A, in such sort that the difference shall be less than the figure G.*

Demonstration. The Square inscribed is more than the one half of the Circle, it being the half of the Square described about the Circle, and in describing the Octagon, you take away more than the half of the Remainder, that is to say, the four Segments C H D, D I E, E K F, C L F. For the Triangle C H D, is the half of the Rectangle C O, (by the 34th. of the 1st.) *it is then more than the half of the Segment C H D; it is the same of the other Arks. In like manner, in describing a Polygon of Sixteen Sides, you take away more than one half of that which remains in the Circle; and so of the rest. You will then at last leave a lesser*

a leſſer quantity than G. For it is evident that having propoſed Two unequal Quantities, if you take away more than the half of the greater; and again more than the half of the remainder, and again more than the half of that remainder, and ſo forward; that which remains ſhall be leſs than the ſecond quantity. Let us ſuppoſe that the ſecond is contained One Hundred times in the firſt; it is evident that by Dividing the firſt into One Hundred parts, in ſuch ſort that the firſt may have a greater Reaſon to the ſecond, than of Two to One; the ſecond ſhall be leſs than the Hundreth part. So then you ſhall at length meet with a Polygon which ſhall be leſs ſurpaſſed by the Circle, than the Circle doth the figure A; that is to ſay, that which remains of the Circle, having taken away the Polygon, ſhall be leſs than G. The Polygon ſhall be greater than the figure A.

PROPOSITION II.

THEOREM.

CIrcles are in the ſame Ratio as are the Squares of their Diameters.

I Demonſtrate that the Circles A and B are in the ſame Ratio as are the Squares

Squares of CD, EF. For if they were not in the ſame Ratio, the Circle A would have a greater Ratio to the Circle B, than the Square of CD to the Square of EF. Let the figure G have the ſame Ratio to the Circle B, as hath the Square of CD to the Square of EF: the figure G ſhall be leſſer than the Circle A; (*by the preceding Lemma*) there may be inſcribed a Regular Polygon greater than G in the Circle A. Let there alſo be inſcribed in the Circle B, a like regular Polygon.

Demonſtration. The Polygon A to the Polygon B, hath the ſame Ratio, as the Square of CD to the Square of EF; that is to ſay, the ſame as hath G to the Circle B: Now the quantity G is leſſer than the Polygon inſcribed in A: ſo then (*by the 14th. of the 5th.*) the Circle ſhould be leſs than the Polygon which is inſcribed, which is evidently falſe. It muſt then be ſaid, that the figure G, leſs than the Circle A, cannot have the ſame Ratio to the Circle B, as the Square of CD to the Square of EF; and by conſequence, that Circle A hath not a greater Ratio to the Circle B, than the Square of CD to the Square of EF. Neither hath it leſs, becauſe

that the Circle B to the Circle A, would have a greater Ratio, and there would be applyed the ſame Demonſtration.

Coroll. 1. Circles are in duplicate Ratio of that of their Diameters; becauſe that Squares being like or ſimilar, are in duplicate Ratio of that of their ſides.

Coroll. 2. Circles are in the ſame Ratio, as are the ſimilar Polygons inſcribed in them.

Coroll. 3. This general Rule muſt be well taken notice of, when like figures inſcribed in other like figures, in ſuch ſort that they become nearer and nearer, and degenerate, in fine, into thoſe figures, they are always in the ſame Ratio. I would ſay, that if like regular Polygons be deſcribed in ſeveral Circles, they are in the ſame Ratio, as are the Squares of the Diameters; and that the greater number of ſides they are made to have, they become ſo much the nearer the Circles; the Circle ſhall have the ſame Ratio as the Squares of their Diameters. This way of meaſuring Bodies by inſcription is very neceſſary.

USE

USE.

THis Propoſition is very univerſal, and is the way of our reaſoning on Circles, after the ſame manner as on Squares. For example, we ſay (in the 47th. of the firſt,) *that in a Rectangular Triangle, that the Square of the Baſe is equal to the Square of the other ſides taken together. We may ſay the ſame of Circles; that is to ſay, that the Circle deſcribed on the Baſe of a Rectangular Triangle is equal to the Circles which hath the ſides for Diameters, and after this we may augment or diminiſh a Circle into what Proportion we liſt. We prove alſo in Opticks, that the Light decreaſeth in duplicate Ratio of that of the diſtance of the Luminous Body.*

PROPOSITION III.

THEOREM.

EVery Pyramid whoſe Baſe is Triangular, may be divided into two equal Priſms, which are greater than half of the Pyramid, and into Two equal Pyramids.

There may be found in the Pyramid

ABCD, two equal Prisms EBFI, EHKC, which shall be greater than half the Pyramid. Divide the six Sides of the Pyramid into equal parts in G, F, E, I, H, K; and draw the Lines EG, GF, FE, HI, IK, EK.

Demonstration. In the Triangle ABD, there is the same Ratio of AG to GB, as of AF to FD; seeing they are equal: thence (*by the 2d. of the 6th.*) GF, BD, are parallels; and GF shall be the half of BD, that is to say, equal to BH. In like manner GE, BI; FE, HI, shall be parallel and equal; and (*by the 15th. of the 11th.*) the Planes GFE, BHI, shall be parallel; and by consequence EBFI, shall be a Prism. I say the same of the figure HEKF, which shall also be a Prism equal to the former; and (*by the 40th. of the 11th.*) seeing the Parallelogram Base HIKD, is double to the Triangle BHI, (*by the 41th. of the first.*)

Secondly, the Pyramids AEFG, ECKI, are like and equal.

Demonstration. The Triangles AFG, FDH, are equal (*by the 3d. of the first,*) as also FDH, EIK. In like manner the Triangles AGE, EIC, and so of the other Triangles of the Pyramid; they

they are then equal (*by the* 10*th. and* 11*th. Def.*) they are alſo ſimilar to the great Pyramid A B D C; for the Triangles A B C, A G E; are like (*by the* 2*d. of the* 6*th.*) the Lines G E being parallel; which I could Demonſtrate in all the Triangles of the leſſer Pyramids.

In fine, I conclude that the Priſms are more than the one half of the firſt Pyramid. For if each were equal to one of the leſſer Pyramids, the Two Priſms would be the half of the great Pyramid: Now they are greater than one of the Pyramids; as the Priſm G H E, contains a Pyramid A G F E, which may eaſily be proved from the paralleliſm of their Sides; whence I conclude that the Two Priſms taken together, are greater than the Two Pyramids, and conſequently greater than half the greater Pyramid.

PROPOSITION IV.

THEOREM.

IF Two Triangular Pyramids of equal height, be divided into Two Priſms and Two Pyramids, and that the laſt Pyramids be divided after the ſame manner, all the Priſms of the one Pyramid ſhall have the ſame Ratio to all thoſe of the other, as the Baſe of the one Pyramid hath to the Baſe of the other.

If one divide the Two Pyramids ABCD, DEFG, of equal height, and of Triangular Baſes, into Two Priſms, and into Two Pyramids, according to the method of the Third Propoſition; and if one ſhould ſubdivide after the ſame manner the Two little Pyramids, and ſo conſecutively, in ſuch ſort that there be as many diviſions in the one as in the other, there then being the ſame number of Priſms in both. I ſay that all the Priſms of the one, to all the Priſms of the other, ſhall be in the ſame Ratio as are the Baſes.

Demonſtration. Seeing the Pyramids are of the ſame height; the Priſms produced by the firſt diviſion, ſhall alſo have

have the ſame height, ſeeing each hath the half of that of their Pyramids: Now Priſms of equal height, are in the ſame Ratio as are their Baſes, (*by the Coroll. of the* 32*d. of the* 11*th.*) the Baſes BTV, EPX, are like unto the Baſes BDC, EGF, and having for Sides the half of thoſe of the great Baſes, they ſhall be but the one fourth of the great Baſes; ſo then they ſhall be in the ſame Ratio as are the great Baſes. Therefore the firſt Priſms ſhall have the ſame Ratio as the greater Baſes. I prove after the ſame manner, that the Priſms produced by the ſecond Diviſion, that is to ſay of the little Pyramids, ſhall be in the ſame Ratio, as the greater Baſes. Therefore all the Priſms of the one, have the ſame Ratio to all the Priſms of the other, as the Baſe to the Baſe.

USE.

THeſe Two Propoſitions were neceſſary to compare Pyramids the one with the other, and to meaſure them.

PROPOSITION V.

THEOREM.

TRiangular Pyramids of the ſame height, are in the ſame Ratio as their Baſes.

The Pyramids A B C D, E F G H, are in the ſame Ratio as are their Baſes. For if they were not, one of the Pyramids, for example A B C D, would have a greater Ratio to the Pyramid E F G H, than the Baſe B C D to the Baſe F G H. So that a quantity L, leſs than A B C D, would have the ſame Ratio to the Pyramid E F G H, as the Baſe B C D to the Baſe F G H. Divide the Pyramid A B C D after the manner of the *Third Propoſition*; divide alſo the Pyramid which will reſult of the firſt diviſion into Two Priſms, and into two Pyramids; and thoſe again into Two other Priſms; continue dividing after this manner as long as need requires: Seeing the Priſms of the firſt diviſion are more than the half of the Pyramid A B C D (*by the 3d.*) and that the Priſms of the ſecond Diviſion is greater than half the Remainder; that is

to

to ſay than the little Pyramid, and that thoſe of the Third Diviſion is more than the half Remainder; it is evident, that by making many ſub-diviſions, there will at length remain a quantity leſs than the exceſs of the Pyramid A B C D above the quantity L; that is to ſay, that all the Priſms being put together, ſhall be greater than the quantity L. Make as many Diviſions in the Pyramid E F G H, untill you have as many Priſms as there are in A B C D.

Demonſtration. The Priſms of A B C D hath the ſame Ratio to the Priſms of E F G H, as hath the Baſe B C D to the Baſe F G H: Now the Ratio of the Baſe B C D to the Baſe F G H, is the ſame with that of the quantity L to the Pyramid E F G H. There is therefore the ſame Ratio of the Priſms of A B C D to the Priſms of E F G H, as of the quantity L to the Pyramid E F G H. Again, the Priſms of A B C D are greater than the quantity L; thence (*by the 14th. of the 5th*) the Priſms comprehended in the Pyramid E F G H, would be greater than the ſame Pyramid E F G H; which is evidently falſe, the part not being capable of being greater than the whole. We muſt then acknowledge

that a Magnitude leſs than one of the Pyramids, cannot have the ſame Ratio to the other, as hath the Baſe to the Baſe; and conſequently, neither of the Pyramids hath a greater Ratio to the other, than the Baſe to the Baſe.

PROPOSITION VI.

THEOREM.

ALL ſorts of Pyramids, of the ſame height, have the ſame Ratio as their Baſes.

The Pyramids ABC, DEFG, of the ſame height, are in the ſame Ratio as are their Baſes BC, EFG. Divide the Baſes into Triangles.

Demonſtration. The Triangular Pyramids AB, DE, of the ſame height, are in the ſame Ratio as are their Baſes (*by the 5th.*) In like manner, the Triangular Pyramids AC, DF, are in the ſame Ratio as their Baſes. There ſhall be then the ſame Ratio of the Pyramid ABC to the Pyramid DEF, as of the Baſe BC to the Baſe EF (*by the 12th. of the 5th.*) Again, ſeeing there is the ſame Ratio of the Pyramid DEF to the Pyramid ABC, as of the Baſe EF to the

the Base B C; and that there is also the same Ratio of the Pyramid D G to the Pyramid A B C, as of the Base G to the Base B C; there will be also the same Ratio of the Pyramid D E F G to the Pyramid A B C, as of the Base E F G to Base B C.

PROPOSITION VII.

THEOREM.

EVery Pyramid is the Third part of a Prism of the same Base, and of the same height.

Let there be proposed in the first place a Triangular Prism A B; I say that a Pyramid which hath for its Base one of the Triangles A C E, B D F, and which shall be of the same height, as the Pyramid A C E F, shall be the Third part of the Prism. Draw the Three Diagonals A F, D C, F C, of the same parallelogram.

Demonstration. The Prism is divided into Three equal Pyramids A C F E, A C F D, C F B D; thence each of them shall be the Third part of the Prism. The Two first, having for Base the Triangles A F E, A F D, which are equal

(*by*

(*by the* 34*th. of the first*) and for height the perpendicular drawn from their top C to the Plane of their Base A F, they shall be equal (*by the preceding.*) The Pyramids A C F D, C F B D, which have for Bases equal Triangles A D C, D C B, and the same Vertex F, shall also be equal (*by the preceding.*) Thence one of these Pyramids, for example A F E C, which hath the same Base A C E, as the Prism; and the same height, which would be the perpendicular drawn from the point F, to the Plane of the Base A C E; is the third part of the same. If the Prism were a Polygon, it must be then divided into several Triangular Prisms; and the Pyramid which would have the same Base, and the same height, would be also divided into so many Triangular Pyramids, each of which would be the Third part of its Prism. Therefore (*by the* 32*d. of the* 5*th.*) the Polygonal Pyramid would be the Third part of the Polygonal Prism.

PRO-

PROPOSITION VIII.

THEOREM.

LIke Pyramids are in triple Ratio of their homologous Sides.

If the Pyramids are Triangular, they may be converted into Prisms, which shall be also like, seeing they would have some Planes, which would be the same as those of the Pyramids. Now like Prisms are in Triple Ratio of their homologous sides, (*by the 39th. of the 11th.*) thence the Pyramids which are the Third of them (*by the preceding*) shall be in Triple Ratio of their homologous sides.

If the Pyramids are Polygons, they may be reduced into Triangular Pyramids.

PROPOSITION IX.

THEOREM.

EQual Pyramids have their height and Bases Reciprocal, and those Pyramids whose height and Bases are reciprocal, are equal.

Let

Let there be propoſed Two Triangular Pyramids, they ſhall have their Baſes and heights reciprocal. Let there be made Priſms of equal heights and Baſes; ſeeing thoſe Priſms are Triple each to its Pyramid (*by the 7th.*) they ſhall be alſo equal. Now equal Priſms, have their Baſes and heights reciprocal (*by the 4th. Coroll. of the 39th.*) thence the Baſes and heights of the Pyramids, which are the ſame with thoſe of the Priſms ſhall be reciprocal to them.

Secondly, if the Baſes and heights of Pyramids be reciprocal, the Priſms ſhall be equal, as alſo the Pyramids, which are the Third parts of them.

If the Pyramids were Polygons, they ought to be reduced into Triangular Pyramids.

Coroll. There might be made ſome other *Propoſitions*; for example, that Pyramids of equal height, are in the ſame Ratio as their Baſes, and thoſe which have the ſame Baſe, are in the ſame Ratio as their heights.

USE

USE

FRom these Propositions is gathered a way of measuring of Pyramids; which is, to multiply the Base by the one third of its height. Then there might be made some other Propositions, as that if a Prism be equal to a Pyramid, the Bases and height of the Prism, with the Third part of the height of the Pyramid shall be reciprocal; that is to say, if there be the same Ratio of the Pyramid to the Base of the Prism, as of the height of the Prism to the third part of the height of the Pyramid; the Pyramid and Prism shall then be equal.

LEMMA.

IF there be proposed a quantity less than a Cylinder, there may be inscribed in that Cylinder a Polygon Prism greater than that quantity.

If the Quantity A be less than the Cylinder which hath the Circle B for its Base; there may be inscribed in the Cylinder a Polygon Prism greater than the quantity A.

A. The Square CDEF is inſcribed, GHIK is circumſcribed, CLDMENFO is an Octogon. Draw the Tangent PLQ, and continue the Sides ED, FC, to P and Q; and imagine as many Priſms of the ſame height as the Cylinder, which have for their Baſes thoſe Polygons. That which hath for its Baſe the Circumſcribed Square incompaſſeth the Cylinder, and that on the inſcribed Square is alſo inſcribed in the Cylinder.

Demonſtration. The Priſms which have the ſame height, are in the ſame Ratio as are their Baſes (by the Coroll. of the 39th. of the 11th.) *and the inſcribed Square being the half of the circumſcribed Square, its Priſm ſhall be the half of the other; it ſhall then be more than the half of the Cylinder. Making a Priſm which hath the Octagon for Baſe, there will be taken away more than the half of that which remained for the Cylinder, having taken away the Priſm of the inſcribed Square; becauſe the Triangle CLD is half the rectangle CQ; and becauſe that Priſms of equal or the ſame height, are in Ratio as are their Baſes, the Priſm which hath for its Baſe the Triangle CLD, ſhall be the half of the Priſm of the Rectangle DCPQ; it ſhall then be more than*

the

the half of the part of the Cylinder, which hath for Base the Segment D L C. It is the same of the other Segments. I demonstrate after the same manner, that in making a Polygon Prism of Sixteen Sides, I take away more than half of what remained of the Cylinder, by taking away the Octogonal Prism; there will remain a part of the Cylinder, less than the excess of the Cylinder above the quantity A. We shall then have a Prism inscribed in the Cylinder, which will be less exceeded by the Cylinder, than the quantity A. One might argue after the same method about inscribed Pyramids in a Cone.

PROPOSITION X.

THEOREM.

A Cone is the Third part of a Cylinder of the same Base, and the same height.

If a Cone, and a Cylinder have the Circle A for Base, and the same height; the Cylinder shall be triple to the Cone. For if the Cylinder had a greater Ratio to the Cone, than the Triple; a quantity B less than the Cylinder, would have the same Ratio to the Cone, as Three

Three to One; and *(by the preceding Lemma)* there might be inſcribed in the Cylinder, a Polygonal Priſm greater than the quantity B. Let us ſuppoſe that that quantity is that which hath for Baſe the Polygon CDEFGH. Make alſo on the ſame Baſe, a Pyramid inſcribed in the Cone.

Demonſtration. The Cylinder, the Cone, the Priſm, and the Pyramid, are all of the ſame height; thence the Priſm is triple to the Pyramid *(by the 7th.)* Now the quantity B is alſo the triple of the Cone; there is then therefore the ſame Ratio of the Priſm to the Pyramid, as of the quantity B to the Cone; and *(by the 14th. of the 5th.)* ſeeing the Priſm is greater than the quantity B, the Pyramid ſhould be greater than the Cone in which it is inſcribed, which cannot be.

But if it were ſaid that the Cone hath a greater Ratio to the Cylinder, than one to three; there might be taken the ſame method to Demonſtrate the contrary.

PRO-

PROPOSITION XI.

THEOREM.

Cylinders and Cones of the ſame height, are in the ſame Ratio as are their Baſes.

There is propoſed two Cylinders or two Cones of the ſame height, which have the Circles A and B for their Baſes: I ſay they are in the ſame Ratio as are their Baſes. For if they are not in the ſame Ratio; one of them, for example A, ſhall have a greater Ratio to the Cylinder B, than the Baſe A to the Baſe B; ſo that a quantity L, leſs than the Cylinder A, would have the ſame Ratio to the Cylinder B, as hath the Baſe A to the Baſe B. There may then be inſcribed a Polygon Priſm in the Cylinder A, greater than the quantity L. Let it be that which hath for Baſe the Polygon CDEF; and let there be inſcribed a like Polygon GHIK, in the Baſe B, which ſerveth for Baſe to the Cylinder of the ſame height.

Demonſtration.

Demonſtration. The Priſms A and B are in the ſame Ratio as are their Polygon Baſes (*by the Coroll. 4. of the 39th. of the 11th.*) and the Polygons are in the ſame Ratio as are the Circles (*by the Coroll. of the 2d.*) ſo then the Priſm A ſhall be in the ſame Ratio to the Priſm B, as the Circle A to the Circle B. Now as the Circle A is to the Cirle B, ſo is the quantity L to the Cylinder B; therefore as the Priſm A is to the Priſm B, ſo is the quantity L to the Cylinder B. The Priſm A is greater than the quantity L; by conſequence (*according to the 14th. of the 5th.*) the Priſm B inſcribed in the Cylinder B, would be greater than it, which cannot be. Therefore neither Cylinder hath greater Ratio to the other, than that of its Baſe to the Baſe of the other.

Coroll. Cylinders are triple to Cones of the ſame height, therefore Cones of the ſame height, are in the ſame Ratio as are their Baſes.

PRO-

PROPOSITION XII.

THEOREM.

Like Cylinders and Cones are in Triple Ratio to that of the Diameters of their Bases.

Let there be proposed two like Cylinders, or two Cones, which have for their Bases the Circles A and B. I say that the Ratio of the Cylinder A to the Cylinder B, is in Triple Ratio of the Diameter DC to the Diameter EF. For if it hath not a Triple Ratio, let the quantity G, less than the Cylinder A, have to the Cylinder B a Triple Ratio of that of the Diameter DC to the Diameter EF; and let there be inscribed in the Cylinder A a Prism, which let be greater than G; and another like thereto in the Cylinder B: they shall have the same Altitude or height as had the Cylinders, for like Cylinders have their heights and the Diameters of their Bases proportional, as well as the Prisms (*by the* 11*th. Def. of the* 11*th.*)

Demonstration. The Diameter DC hath the same Ratio to the Diameter EF,

EF, as hath the Side DI to the Side DL, or DC, to EF, (*as I Demonſtrated in the firſt.*) Now the Priſms are in Triple Ratio of their homologous Sides (*by the Corell. 4. of the 39 of 11.*) therefore the Priſm A to the Priſm B, is in Triple Ratio of DC to EF. We have ſuppoſed that the quantity G, in reſpect of the Cylinder B, was in Triple Ratio of the Diameter DC to the Diameter EF. So then there will be the ſame Ratio of the Priſm A to the Priſm B, as of the quantity G to the Cylinder B; and (*by the 14th. of the fifth*) ſeeing that the Priſm A, is greater than the quantity G; the Priſm B, that is to ſay, deſcribed in the Cylinder B, would be greater than the ſame Cylinder, which is impoſſible. Therefore like Cylinders are in Triple Ratio of the Diameters of their Baſes.

Cones are the Third parts of Cylinders (*by the Tenth*;) therefore like Cones are in Triple Ratio of that of the Diameters of their Baſes.

PRO-

PROPOSITION XIII.

THEOREM.

IF a Cylinder be cut by a Plane parallel to its Base, the parts of the Axis shall be in the same Ratio, as are the parts of the Cylinder.

Let the Cylinder A B, be cut by the Plane D C, parallel to its Base. I say that there shall be the same Ratio of the Cylinder A F to the Cylinder F B, as of the Line A F to the Line F B. Draw the Line B G perpendicular to the Plane of its Base A; draw also in the Planes of the Circles D C, and A, the Lines F E, A G.

Demonstration. The Plane of the Triangle B A G, cutteth the parallel Planes A and D C; thence the Sections E F, and A G, are parallel (*by the* 16*th. of the* 11*th.*) So then there is the same Ratio of A F to F B, as of the height G E to E B. Let there be taken an Aliquot part of E B, and having divided G E, E B, into parts equal to that Aliquot part, let there be drawn Planes parallel to the Base A G: You shall have so many Cilinders of the same height, which

having their Baſes and heights equal, they ſhall be equal (*by the* 11*th.*) Again, the Lines A F and F B, ſhall be divided after the ſame manner as E G, E B, (*by the* 16*th. of the firſt*) So then the Line A F contains as many times the Aliquot part of the Line F B, as the Cylinder A F contains a like Aliquot parts of F B. There is thence the ſame Ratio of the parts of the Cylinder, as of the parts of the Axis.

Coroll. The parts of the perpendicular are in the ſame Ratio as are the parts of the Cylinder.

PROPOSITION XIV.

THEOREM.

Cylinders and Cones having the ſame Baſe, are in the ſame Ratio as are their height.

Two Cylinders A B, C D, of equal Baſes, being propoſed; cut out of the greater a Cylinder of the ſame height, with the leſſer, by drawing a Plane E F parallel to its Baſe. It is evident that the Cylinders E F, A B, are equal (*by the* 11*th.*) and that C F to C D, hath the

the ſame Ratio as G I to G H, or (*by the Coroll. of the preceding*) as the height of the Cylinder C F to the height of C D, there is thence the ſame Ratio of A B to C D, as of the height of E F, or A B, to the height of C D.

As to Cones, ſeeing they are the one thirds of Cylinders, if they have equal Baſes, they ſhall be alſo in the ſame Ratio as are their height.

PROPOSITION XV.

THEOREM.

EQual Cylinders and Cones, have their Baſes and height reciprocal, and thoſe which have their Baſes and heights reciprocal, are equal.

If the Cylinders A B, C D, are equal; there ſhall be the ſame Ratio of the Baſe B to the Baſe D, as of the height C D to the height A B. Let the height D E be equal to the height A B.

Demonſtration. There is the ſame Ratio of the Cylinder A B to the Cylinder D E, of the ſame height, as of the Baſe B to the Baſe D (*by the 14th.*) Now as the Cylinder A B is to the Cylinder D E; ſo is the Cylinder C D

equal to AB, to the Cylinder DE; that is to ſay, ſo is the height CD to the height AB or DE. Therefore as the Baſe B is to the Baſe D, ſo is the height CD to the height AB.

Secondly, If there be the ſame Ratio of the Baſe B to the Baſe D, as of the height CD to the height AB; the Cylinders AB, CD, ſhall be equal. For the Cylinder AB is to the Cylinder DE, as is the Baſe B to the Baſe D, and the Cylinder CD ſhall have the ſame Ratio to the Cylinder DE, as CD to DE; there ſhall thence be the ſame Ratio of AB to DE, as of CD to DE; and (*by the 8th. of the 5th.*) the Cylinders AB, and CD, ſhall be equal.

The Propoſitions 16 *and* 17 *are very difficult, and are only of uſe in the* 18*th. which I will Demonſtrate after a more eaſie method by the Two following Lemma's.*

LEM-

LEMMA I.

IF there be propoſed a quantity leſs than a Sphere; there may be inſcribed in the ſame Sphere a number of Cylinders, which taken together ſhall be of the ſame height, and greater than that quantity.

Let ABC be half of a great Circle of the Sphere, and let the quantity D be leſs than the ſame hemi ſphere; I ſay that there may be inſcribed therein ſeveral Cylinders of the ſame height, which taken together, ſhall be greater than the quantity D. For if the hemi-ſphere ſurpaſſeth in bigneſs the quantity D, it will ſurpaſs it by a certain quantity; let that quantity be the Cylinder MP. In ſuch ſort that the quantity D, and MP taken together, be equal to the hemi-ſphere. Make or conceive it ſo made, that there be the ſame Ratio of a great Circle of the Sphere to the Baſe MO, as of the height MN to the height R. Divide the Line EB into as many equal parts as you pleaſe, each of them leſs than R; and drawing parallels to the Line AG, deſcribe inſcribed and circumſcribed parallelograms. The Number of the circumſcribed ſhall ſurpaſs by a unit thoſe of the inſcribed. Lemma. Fig. I.

Now all the circumſcribed rectangles ſurpaſs the inſcribed, by the little Rectangles through which the Circumference of the Circle paſſeth; and all thoſe Rectangles taken together are equal to the Rectangle A L. Imagine that the Semi-Circle is made to roul about the Diameter E B; the Semi-Circle ſhall deſcribe a hemi-ſphere, and the inſcribed Rectangles will deſcribe inſcribed Cylinders in the ſemi-ſphere, and the Circumſcribed will deſcribe other Cylinders.

Demonſtration. The Circumſcribed Cylinders ſurpaſs more the inſcribed, than doth the hemi-ſphere ſurpaſs the ſame inſcribed Cylinders; ſeeing that they are comprehended within the Circumſcribed Cylinders. Now the Circumſcribed ſurpaſs the inſcribed by ſo much as is the Cylinder A L; therefore the hemi-ſphere ſhall ſurpaſs by leſs the inſcribed Cylinders, than doth the Cylinder made by the Rectangle A L. The Cylinder A L, is leſs than the Cylinder M P; for there is the ſame Ratio of a great Circle of the Sphere which ſerveth for Baſe to the Cylinder A L, as of M P to R; ſo then by the foregoing, *a Cylinder which hath for Baſe a great Circle of the Sphere, and the Altitude R, would be equal to the Cylinder M P.*

Conſe-

Conſequently the hemi-ſphere which ſurpaſſeth the quantity D, by the Cylinder M P; and the inſcribed Cylinders by a quantity leſs than A L, ſurpaſſeth the inſcribed Cylinders by leſs than the quantity D. Therefore the quantity D, is leſs than the inſcribed Cylinders.

What I have ſaid of a hemi ſphere, may be ſaid of a whole Sphere.

LEMMA II.

LIke Cylinders inſcribed in Two Spheres, are in Triple Ratio of the Diameters of the Spheres.

If the two like Cylinders C D, E F, be inſcribed in the Spheres A, B; they ſhall be in Triple Ratio of the Diameters L M, N O. Draw the Lines G D, I F. Lemma. Fig. II.

Demonſtration. The like Right Cylinders C D, E F, are like; ſo then there is the ſame Ratio of H D to D R, as of Q F to F S; as alſo the ſame Ratio of K D to K G, as of P F to P I. And conſequently the Triangles G D K, I F P, are like (by the 6th. of the 6th.) *ſo there ſhall be the ſame Ratio of K D to P F, as of G D to I F, or of L M to O N. Now the like Cylinders C D, E F, are in Triple*

Ratio of K D and P F, the Diameters of their Baſes (by the 12th.) *therefore the like Cylinders C D, E F, inſcribed in the Spheres A and B, are in triple Ratio of the Diameters of the Spheres.*

PROPOSITION XVIII.

THEOREM.

SPheres are in triple Ratio of their Diameters.

The Spheres A and B are in Triple Ratio of their Diameters C D, E F. For if they be not in Triple Ratio; one of the Spheres, as A, ſhall be in a greater Ratio than Triple, of that of C D to E F; therefore a quantity G leſs than the Sphere A, ſhall be in Triple Ratio of that of C D to E F; and ſo one might (*according to the firſt Lemma*) inſcribe in the Sphere A, Cylinders of the ſame height, greater than the quantity G.

Let there be inſcribed in the Sphere B, as many like Cylinders as thoſe of the Cylinder A.

Demonſtration. The Cylinders of the Sphere A, to thoſe of the Sphere B, ſhall be in Triple Ratio of that of C D to E F (*by the preceding*:) Now the quantity G in

in reſpect of the Sphere B, is in Triple Ratio of CD to EF; there is then the ſame Ratio of the Cylinders of the Sphere A, to the like Cylinders of the Sphere B. So then the Cylinders of A being greater than the quantity G; the Cylinder B, that is to ſay, inſcribed in the Sphere B, would be greater than the Sphere B, which is impoſſible. Therefore the Spheres A and B are in Triple Ratio of that of their Diameters.

Coroll. Spheres are in the ſame Ratio as are the Cubes of their Diameters; ſeeing that the Cubes being like ſolids, are in Triple Ratio of that of their Sides.

FINIS.

ERRATA.

ERRATA.

PAge 14. *Line* 21. *read* AFD. *p.* 23. *l.* 17. *for* DF *r.* EF. *l.* 26. *r.* DFE. *l.* 27. *r.* FD. *p.* 24. *l.* 1 *and* 9. *for* FB *r.* FD. *p.* 26. *l.* 8. *r.* HI. *p.* 52. *l.* 14. *r.* ACB. *p.* 55. *l.* 22. *r.* ACF. *p.* 71. *l.* 3. *r.* ACB. *p.* 82. *l.* 19. *r.* ABC. *p.* 117. *l.* 13. *r.* GFE. *p.* 125. *l.* 9. *r.* AD. *p.* 178. *l.* 16. *r.* CFD. *p.* 194. *l. ult. r.* AF. *p.* 197. *l.* 2. *r.* CDA. *p.* 218. *l.* 7. *r.* $\frac{1}{3}$ *and* $\frac{1}{3}$. *p.* 268. *l.* 24. *r.* CE. *p.* 281. *l.* 3. *r.* ECD. *p.* 289. *l.* 4. *r.* are ſimular. *p.* 294. *l.* 13. *r.* eight $\frac{3}{4}$. *p.* 309. *l.* 15. *r.* DBE.

ADVER-

Advertiſement of Globes, Books, Maps, &c. made and ſold by Philip Lea *at the Sign of the* Atlas *and* Hercules *in the* Poultry, *near* Cheapſide, London.

1. A New Size of Globes about 15 Inches Diameter, made according to the more accurate obſervation and diſcoveries of our Modern Aſtronomers and Geographers, and much different from all that ever were yet extant; all the Southern Conſtellations according to Mr. *Hally's* obſervations in the Iſland of St. *Helena*, and many of the Nothern. *Price Four Pounds.*

2. A ſize of Globes of about Ten Inches Diameter, very much Corrected. *Price* 50 *s.*

3. Concave Hemiſpheares Three Inches Diameter, which ſerves as a Caſe for a Terreſtial Globe, and may be carried in the Pocket or fitted up in Frames. *Price* 15 *or* 20 *s.*

4. Another Globe of about Four Inches Diameter, fitted to move in Circular Lines of Braſs, for Demonſtrating the Reaſon of Dyalling, or being erected upon a ſmall Pedeſtal, and ſet North and South, will ſhew the hour of the day by its own ſhadow, or by the help of a moving Meridian, will ſhew the hour of the Day in all parts of the World, *&c.*

14. There is in the Press a Particular description of the general use of Quadrants, for the easie resolving Astronomical, Geometrical, and Gnomonical Problems, and finding the hour and Azimuth universally, *&c.* whereunto is added the use of the Nocturnal and equinoctial Dial.

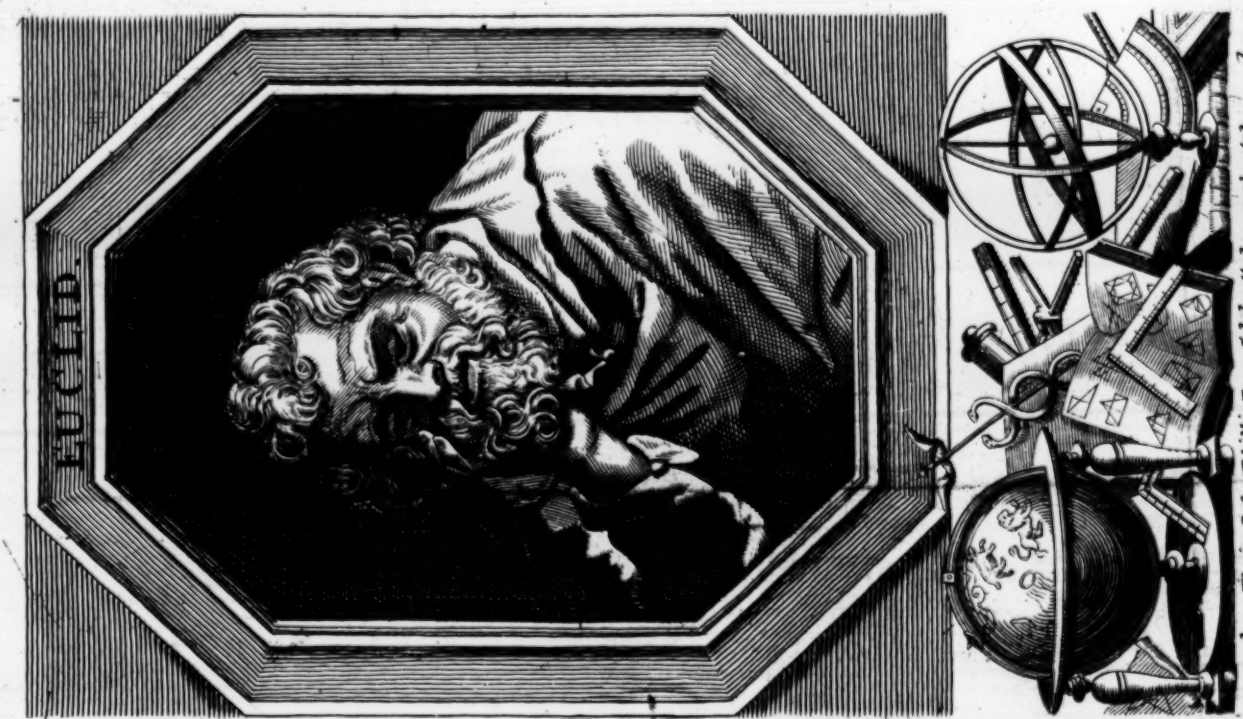
15. A new Map of *England*, *Scotland*, and *Ireland*, with the Roads, and a delineation of the Genealogy of the Kings thereof, from *William* the Conquerer to this present time, with an Alphabetical Table for the ready finding of the places. *Price* 18 *d.*

16. *The Elements of Euclide* explained and demonstrated after a new and most easie method, with the uses of each Proposition, in all the parts of the Mathematicks; by *Claude Francois Millet Dechales*, a Jesuite.

17. New Maps of the World, four quarters, and of all the Countries, and of all sizes, made according to the late discoveries extant, may be had pasted upon Cloath and Coloured, also Sea Plats Mathematical Projection, Books and Instruments whatsoever, are made and sold by *Philip Lea.*

FINIS.

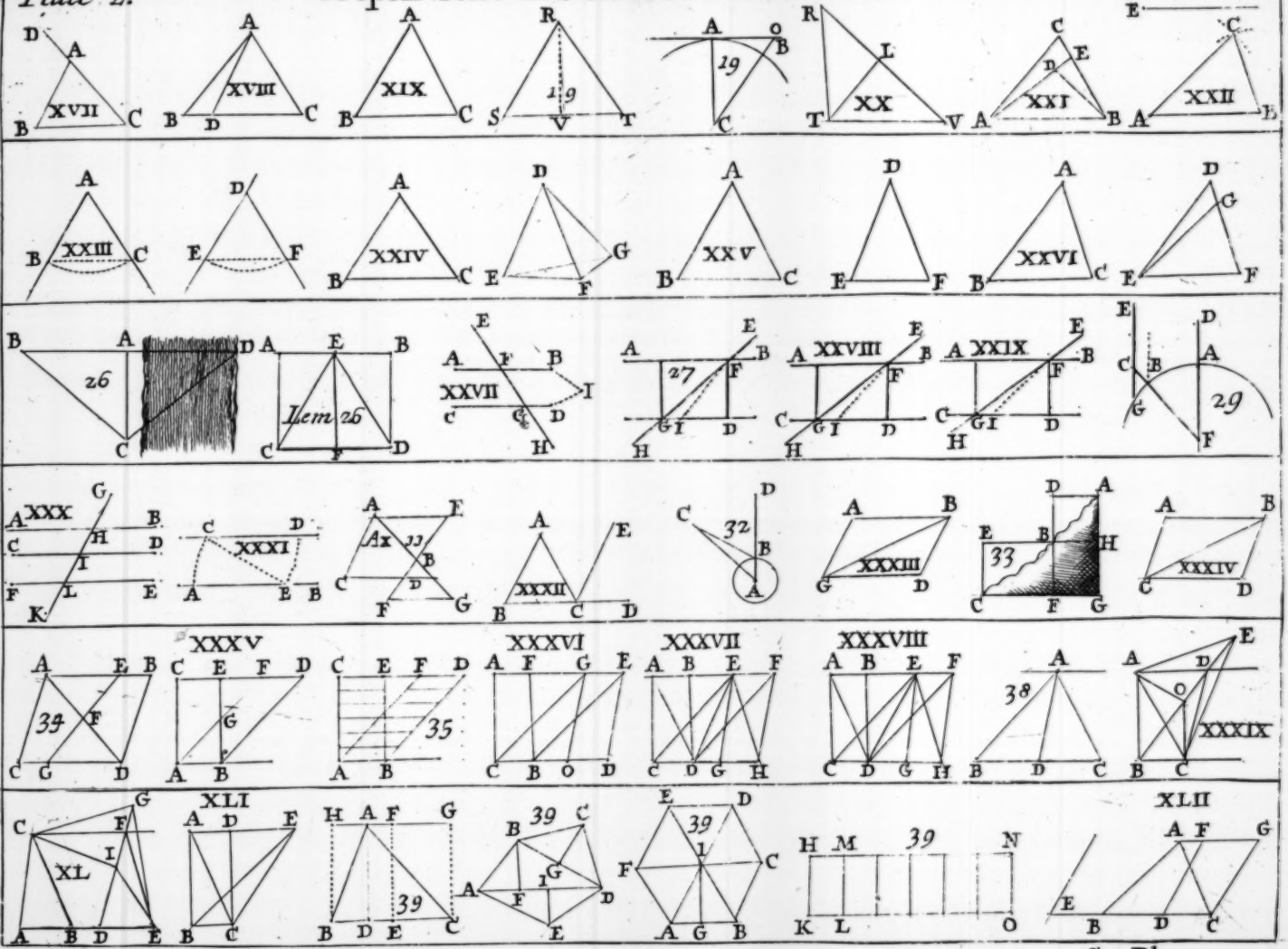
EUCLIDS ELEMENTS



London Printed for Philip Lea Globe Maker at the Atlas and Hercules in Cheapside Near Fryday Street where all Sorts of Globes Spheres Maps Sea-plats Mathematical Books and Instruments are Made and Sold

EUCLIDS ELEMENTS with y^e Uses

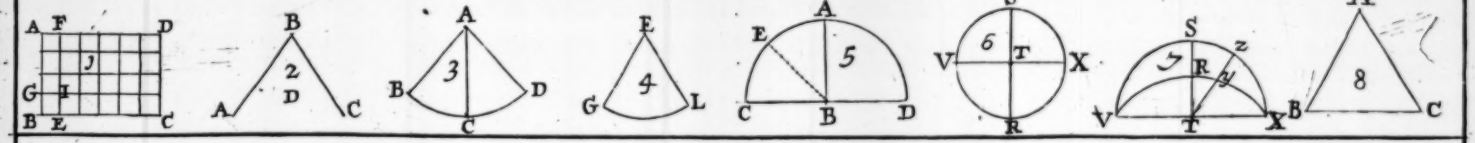
Plate 2. Propositions and Uses of the first Book

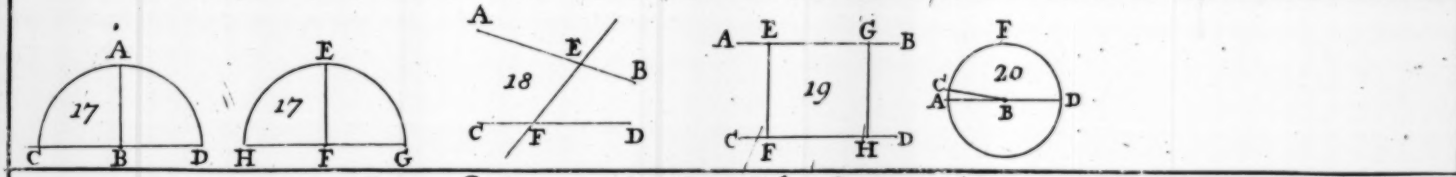


See Plate 3.

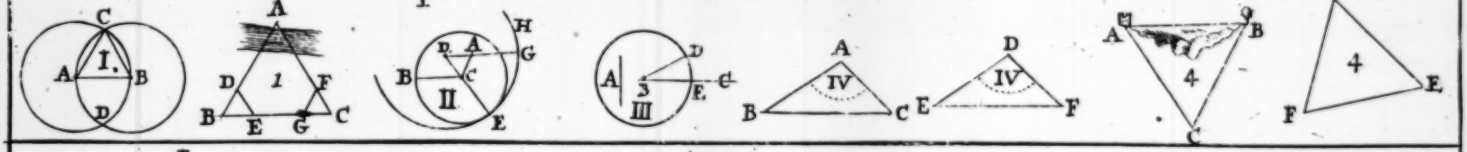
Plate 1.

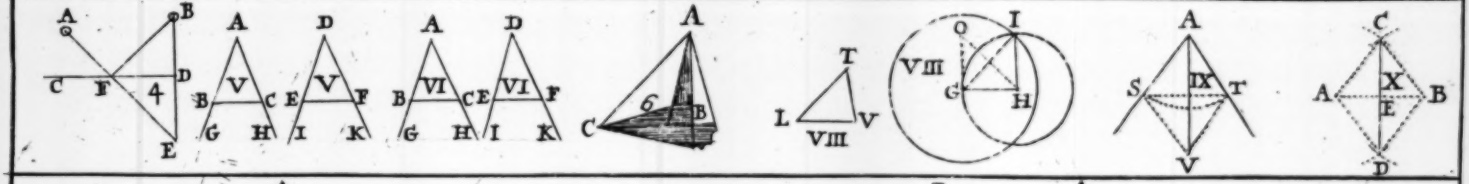
Definitions of the first Book

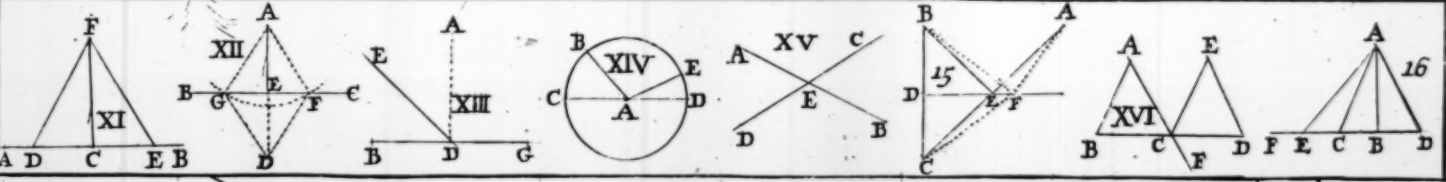




Propositions and Uses of the first Book





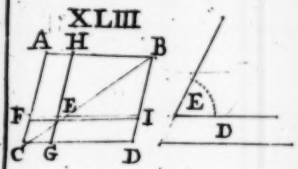


See plate 2.

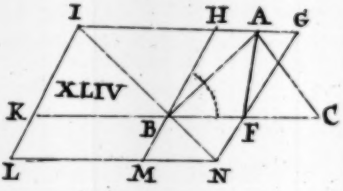
Propositions & Uses of ye first Book

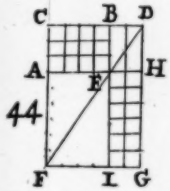
Plate 3.

XLIII

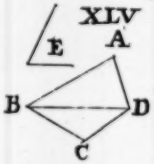


XLIV



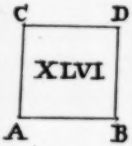


XLV

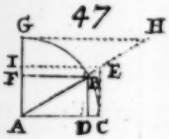


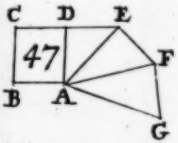
XLV

XLVI



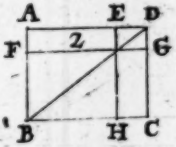
XLVII

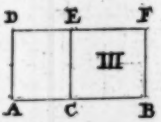


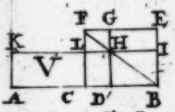


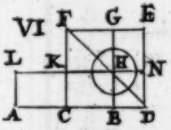
XLVIII

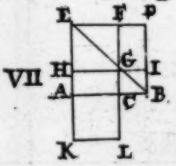
Definitions Propositions & Uses of ye Second Book

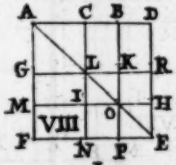


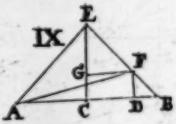


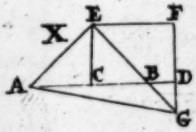


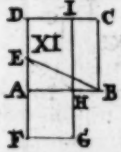


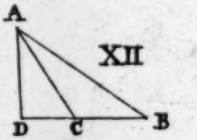


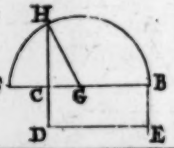




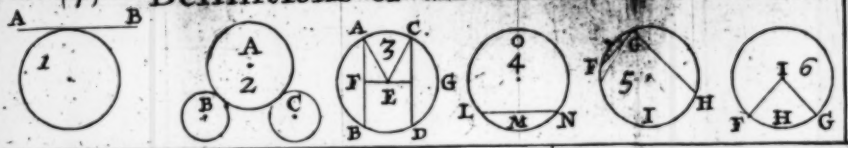






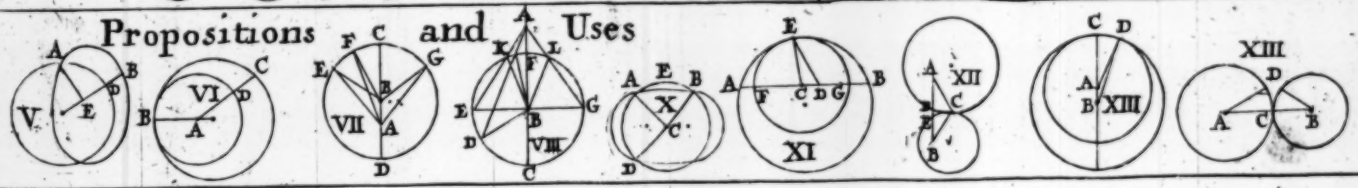


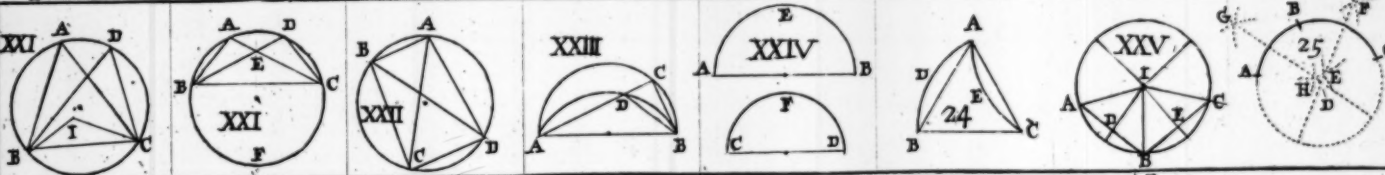
Plate(4) Definitions of the third Book.

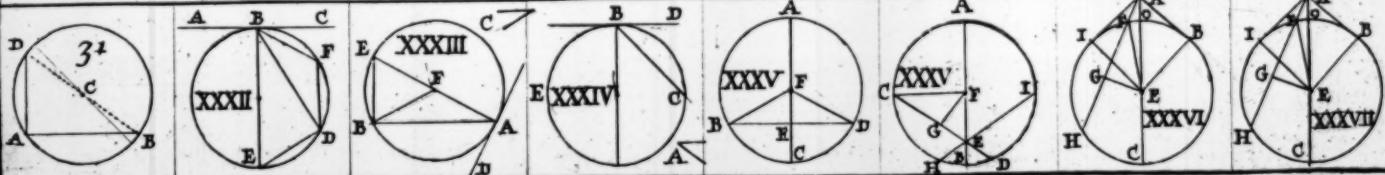


Propositions.

Propositions and Uses



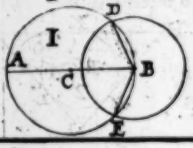




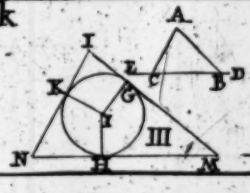
UMI

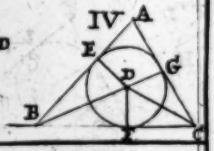
Definitions

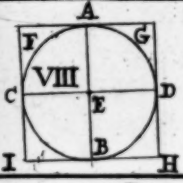
Propsitions of

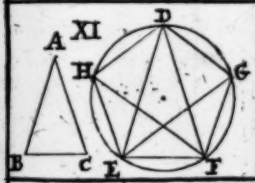


the fouth Book









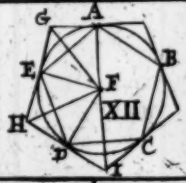
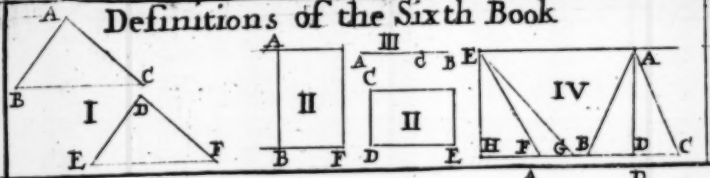
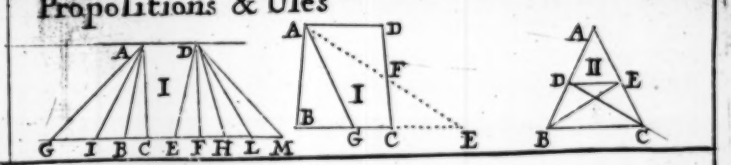


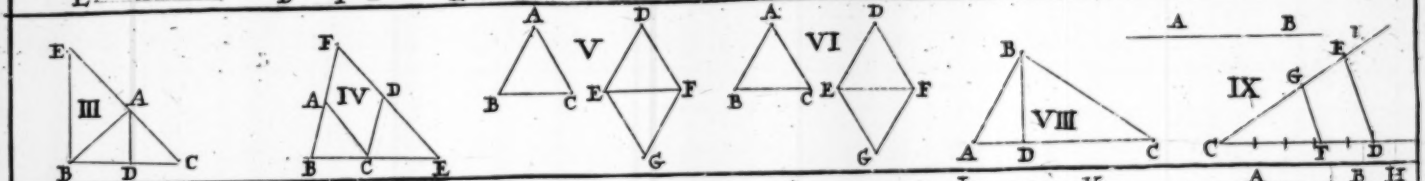
Plate 5.

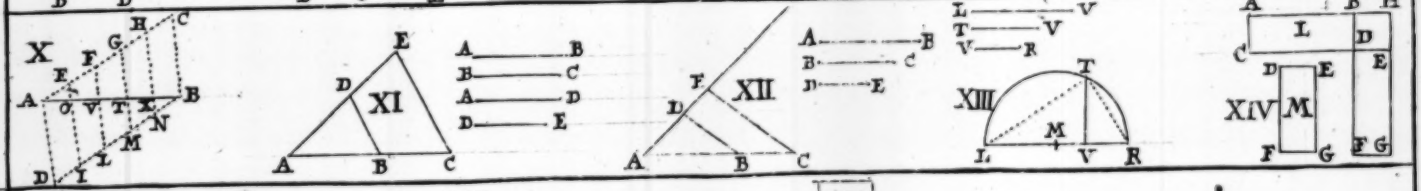
Definitions of the Sixth Book

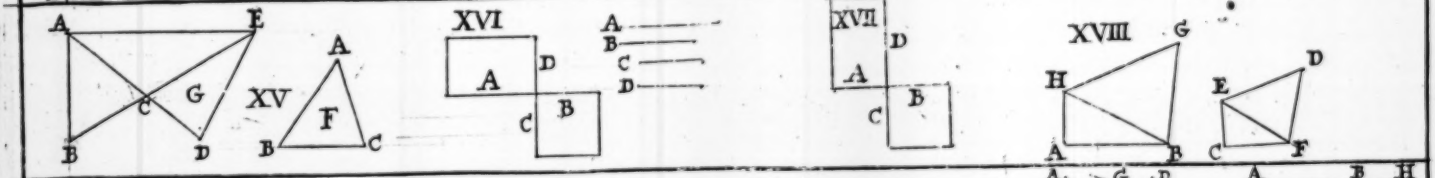


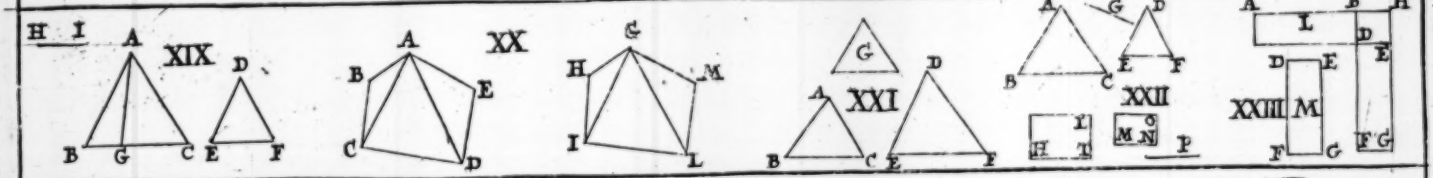
Propositions & Uses











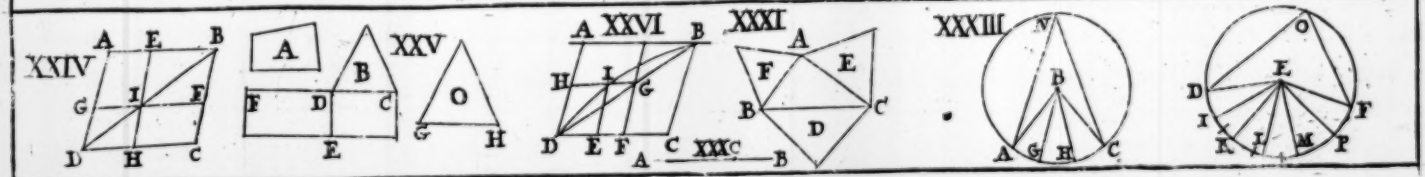


Plate 6.

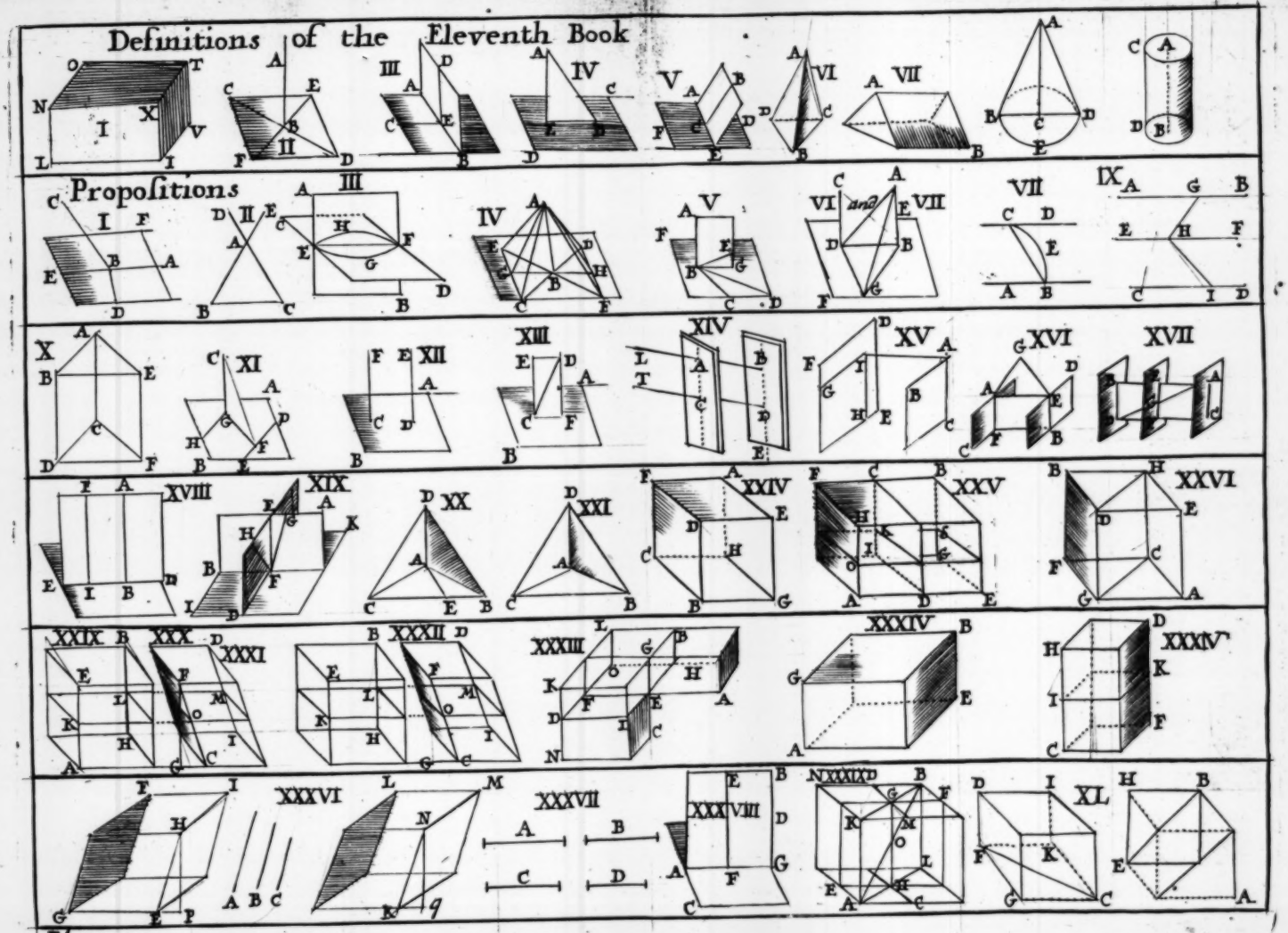
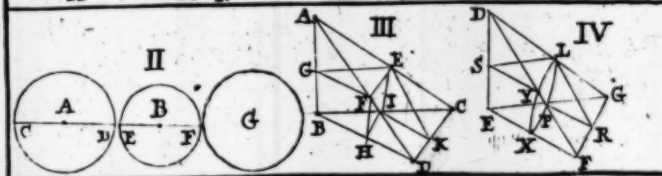
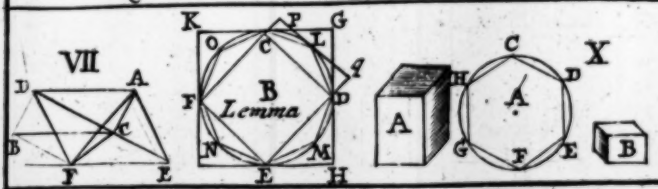
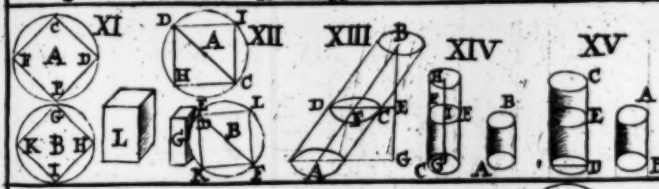

Definitions of the Eleventh Book
I
II
III
IV
V
VI
VII
Propositions
I
II
III
IV
V
VI and VII
VII
IX
X
XI
XII
XIII
XIV
XV
XVI
XVII
XVIII
XIX
XX
XXI
XXII
XXIII
XXIV
XXV
XXVI
XXVII
XXVIII
XXIX
XXX
XXXI
XXXII
XXXIII
XXXIV
XXXV
XXXVI
XXXVII
XXXVIII
XXXIX
XL

Plate 7.

Pro(A)=posi: I of the Twelfth Book







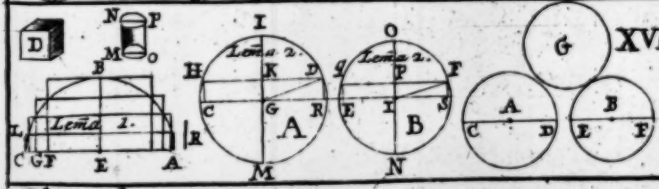


Plate 8.